Exercise 2.1

Question 1:

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\sin^{-1}\left(-\frac{1}{2}\right)=y.$ Then $\sin y = -\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2}.$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}.$

Question 2:

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y.$ Then, $\cos y=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0,\pi]$ and $\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

Question 3:

Find the principal value of cosec^{-1} (2)

Answer

Let $\text{cosec}^{-1}(2) = y$. Then, $\text{cosec}\, y = 2 = \text{cosec}\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Therefore, the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

Question 4:

Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Answer

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$. Then, $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $-\sqrt{3}$.

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

Question 5:

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 6:

Find the principal value of $\tan^{-1}(-1)$

Answer

Let $\tan^{-1}(-1) = y$. Then, $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1$.

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Question 7:

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Answer

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$. Then, $\sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\sec^{-1}$ is

$[0,\pi] - \left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$.

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:

Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Answer

Let $\cot^{-1}\left(\sqrt{3}\right) = y$. Then, $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\cot^{-1}$ is (0,π) and

$$\cot\left(\frac{\pi}{6}\right)=\sqrt{3}.$$

Therefore, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

Question 9:

Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Answer

$$\text{Let } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=y. \text{ Then, } \cos y=-\frac{1}{\sqrt{2}}=-\cos\left(\frac{\pi}{4}\right)=\cos\left(\pi-\frac{\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right).$$

We know that the range of the principal value branch of $\cos^{-1}$ is [0,π] and

$$\cos\left(\frac{3\pi}{4}\right)=-\frac{1}{\sqrt{2}}.$$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

Find the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$

Answer

$$\text{Let } \text{cosec}^{-1}\left(-\sqrt{2}\right)=y. \text{ Then, } \text{cosec } y=-\sqrt{2}=-\text{cosec}\left(\frac{\pi}{4}\right)=\text{cosec}\left(-\frac{\pi}{4}\right).$$

We know that the range of the principal value branch of cosec^{-1} is

$$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\} \text{ and } \text{cosec}\left(-\frac{\pi}{4}\right)=-\sqrt{2}.$$

Therefore, the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

Question 11:

Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

Answer

Let $\tan^{-1}(1)=x$. Then, $\tan x=1=\tan\frac{\pi}{4}$.

$\therefore \tan^{-1}(1)=\frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right)=y$. Then, $\cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right)=z$. Then, $\sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$

$=\frac{3\pi+8\pi-2\pi}{12}=\frac{9\pi}{12}=\frac{3\pi}{4}$

Question 12:

Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Answer

$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x. \text{ Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right).$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = y. \text{ Then, } \sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right).$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

Question 13:

Find the value of if $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$ **(B)** $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$ **(D)** $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Answer

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Question 14:

Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) π **(B)** $-\frac{\pi}{3}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{2\pi}{3}$

Answer

Let $\tan^{-1}\sqrt{3} = x$. Then, $\tan x = \sqrt{3} = \tan\frac{\pi}{3}$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\frac{2\pi}{3}$.

We know that the range of the principal value branch of $\sec^{-1}$ is $[0,\pi] - \left\{\frac{\pi}{2}\right\}$.

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$

Hence, $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$