Exercise 5.4

1. Differentiate the following w.r.t. x:

$\frac{e^x}{\sin x}$

Solution:

Given expression is $\frac{e^x}{\sin x}$

Let $y = \frac{e^x}{\sin x}$ therefore,

$\frac{dy}{dx} = \frac{e^x.\frac{d}{dx}\sin x - \sin x\frac{d}{dx}e^x}{\sin^2 x} = \frac{e^x.\cos x - \sin x.e^x}{\sin^2 x} = \frac{e^x(\cos x - \sin x)}{\sin^2 x}$

2. $e^{\sin^{-1} x}$

Solution:

Given expression is $e^{\sin^{-1} x}$

Let $y = e^{\sin^{-1} x}$, therefore,

$\frac{dy}{dx} = e^{\sin^{-1} x}.\frac{d}{dx}\sin^{-1} x = e^{\sin^{-1} x}.\frac{1}{\sqrt{1-x^2}} = \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}.$

3. e^{x^3}

Solution:

Given expression is e^{x^3}

Let $y = e^{x^3}$, therefore,

$\frac{dy}{dx} = e^{x^3}.\frac{d}{dx}x^3 = e^{x^3}.3x^2 = 3x^2e^{x^3}$

4. $\sin(\tan^{-1} e^{-x})$

Solution:

Given expression is $\sin(\tan^{-1} e^{-x})$

Let $y = \sin(\tan^{-1} e^{-x})$, therefore,

$\frac{dy}{dx} = \cos(\tan^{-1} e^{-x}).\frac{d}{dx}\tan^{-1} e^{-x} = \cos(\tan^{-1} e^{-x}).\frac{1}{1+(e^{-x})^2}.\frac{d}{dx}e^{-x}$

$= \cos(\tan^{-1} e^{-x}).\frac{1}{1+e^{-2x}}.(-e^{-x}) = -\frac{e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}.$

5. $\log(\cos e^x)$

Solution:

Given expression is $\log(\cos e^x)$

Let $y = \log(\cos e^x)$,

Therefore,

$\frac{dy}{dx} = \frac{1}{\cos e^x}.\frac{d}{dx}\cos e^x = \frac{1}{\cos e^x}(-\sin e^x)\frac{d}{dx}e^x = -\tan e^x . e^x$

6. $e^x + e^{x^2} + \cdots + e^{x^5}$

Solution:

Given expression is $e^x + e^{x^2} + \cdots + e^{x^5}$

Let $y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$, therefore,

$\frac{dy}{dx} = e^x + e^{x^2}\frac{d}{dx}x^2 + e^{x^3}\frac{d}{dx}x^3 + e^{x^4}\frac{d}{dx}x^4 + e^{x^5}\frac{d}{dx}x^5$

$= e^x + e^{x^2}.2x + e^{x^3}.3x^2 + e^{x^4}.4x^3 + e^{x^5}.5x^4$

$= e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

7. $\sqrt{e^{\sqrt{x}}}, x > 0$

Solution:

Given expression is $\sqrt{e^{\sqrt{x}}}, x > 0$

Let $y = \sqrt{e^{\sqrt{x}}}$

Therefore,

$$\frac{dy}{dx} = \frac{1}{2\sqrt{e^{\sqrt{x}}}}\frac{d}{dx}e^{\sqrt{x}} = \frac{1}{2\sqrt{e^{\sqrt{x}}}}.e^{\sqrt{x}}.\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{e^{\sqrt{x}}}}.e^{\sqrt{x}}.\frac{1}{2\sqrt{x}} = \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$$

8. $\log(\log x), x > 1$

Solution:

Given expression is $\log(\log x), x > 1$

Let $y = \frac{e^x}{\sin x}$

Therefore,

$$\frac{dy}{dx} = \frac{1}{\log x}.\frac{d}{dx}\log x = \frac{1}{\log x}.\frac{1}{x} = \frac{1}{x\log x}$$

9. $\frac{\cos x}{\log x}, x > 0$

Solution:

Given expression is $\frac{\cos x}{\log x}, x > 0$

Let $y = \frac{\cos x}{\log x}$

Therefore, $\frac{dy}{dx} = \frac{\log x\frac{d}{dx}\cos x - \cos x\frac{d}{dx}\log x}{(\log x)^2} = \frac{\log x.(-\sin x) - \cos x.\frac{1}{x}}{(\log x)^2} = \frac{-(x\sin x\log x + \cos x)}{x(\log x)^2}$

10. $\cos(\log x + e^x)$

Solution:

Given expression is $\cos(\log x + e^x)$

Let $y = \cos(\log x + e^x)$

Therefore,

$\frac{dy}{dx} = -\sin(\log x + e^x).\frac{d}{dx}(\log x + e^x) = -\sin(\log x + e^x).\left(\frac{1}{x} + e^x\right)$