

## Chapter - 5

### Continuity and Differentiability

Class 12

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#### Exercise 5.6

##### Question 1:

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = 2at^2, y = at^4$$

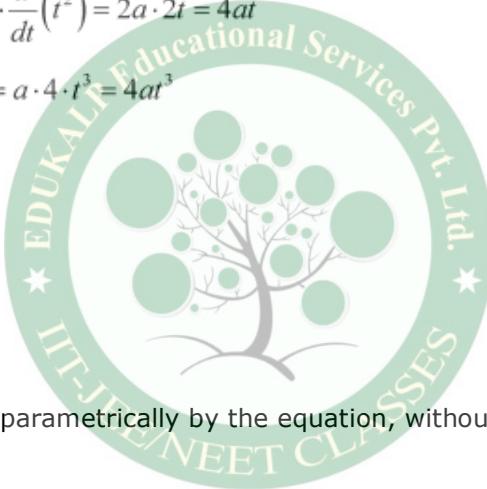
Answer

The given equations are  $x = 2at^2$  and  $y = at^4$

$$\text{Then, } \frac{dx}{dt} = \frac{d}{dt}(2at^2) = 2a \cdot \frac{d}{dt}(t^2) = 2a \cdot 2t = 4at$$

$$\frac{dy}{dt} = \frac{d}{dt}(at^4) = a \cdot \frac{d}{dt}(t^4) = a \cdot 4 \cdot t^3 = 4at^3$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{4at^3}{4at} = t^2$$



##### Question 2:

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = a \cos \theta, y = b \cos \theta$$

Answer

The given equations are  $x = a \cos \theta$  and  $y = b \cos \theta$

$$\text{Then, } \frac{dx}{d\theta} = \frac{d}{d\theta}(a \cos \theta) = a(-\sin \theta) = -a \sin \theta$$

$$\frac{dy}{d\theta} = \frac{d}{d\theta}(b \cos \theta) = b(-\sin \theta) = -b \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-b \sin \theta}{-a \sin \theta} = \frac{b}{a}$$

**Question 3:**

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = \sin t, y = \cos 2t$$

Answer

The given equations are  $x = \sin t$  and  $y = \cos 2t$

$$\text{Then, } \frac{dx}{dt} = \frac{d}{dt}(\sin t) = \cos t$$

$$\frac{dy}{dt} = \frac{d}{dt}(\cos 2t) = -\sin 2t \cdot \frac{d}{dt}(2t) = -2 \sin 2t$$

$$\therefore \frac{dy}{dx} = \frac{\left( \frac{dy}{dt} \right)}{\left( \frac{dx}{dt} \right)} = \frac{-2 \sin 2t}{\cos t} = \frac{-2 \cdot 2 \sin t \cos t}{\cos t} = -4 \sin t$$

**Question 4:**

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = 4t, y = \frac{4}{t}$$

Answer

$$x = 4t \text{ and } y = \frac{4}{t}$$

The given equations are



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$$\frac{dx}{dt} = \frac{d}{dt}(4t) = 4$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t}\right) = 4 \cdot \frac{d}{dt}\left(\frac{1}{t}\right) = 4 \cdot \left(-\frac{1}{t^2}\right) = -\frac{4}{t^2}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(-\frac{4}{t^2}\right)}{4} = -\frac{1}{t^2}$$

#### Question 5:

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = \cos \theta - \cos 2\theta, y = \sin \theta - \sin 2\theta$$

Answer

The given equations are  $x = \cos \theta - \cos 2\theta$  and  $y = \sin \theta - \sin 2\theta$

$$\begin{aligned} \text{Then, } \frac{dx}{d\theta} &= \frac{d}{d\theta}(\cos \theta - \cos 2\theta) = \frac{d}{d\theta}(\cos \theta) - \frac{d}{d\theta}(\cos 2\theta) \\ &= -\sin \theta - (-2 \sin 2\theta) = 2 \sin 2\theta - \sin \theta \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\theta} &= \frac{d}{d\theta}(\sin \theta - \sin 2\theta) = \frac{d}{d\theta}(\sin \theta) - \frac{d}{d\theta}(\sin 2\theta) \\ &= \cos \theta - 2 \cos 2\theta \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{\cos \theta - 2 \cos 2\theta}{2 \sin 2\theta - \sin \theta}$$

#### Question 6:

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = a(\theta - \sin \theta), y = a(1 + \cos \theta)$$

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Answer

The given equations are  $x = a(\theta - \sin \theta)$  and  $y = a(1 + \cos \theta)$

$$\text{Then, } \frac{dx}{d\theta} = a \left[ \frac{d}{d\theta}(\theta) - \frac{d}{d\theta}(\sin \theta) \right] = a(1 - \cos \theta)$$

$$\frac{dy}{d\theta} = a \left[ \frac{d}{d\theta}(1) + \frac{d}{d\theta}(\cos \theta) \right] = a[0 + (-\sin \theta)] = -a \sin \theta$$

$$\therefore \frac{dy}{dx} = \frac{\left( \frac{dy}{d\theta} \right)}{\left( \frac{dx}{d\theta} \right)} = \frac{-a \sin \theta}{a(1 - \cos \theta)} = \frac{-2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = \frac{-\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} = -\cot \frac{\theta}{2}$$

Question 7:

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

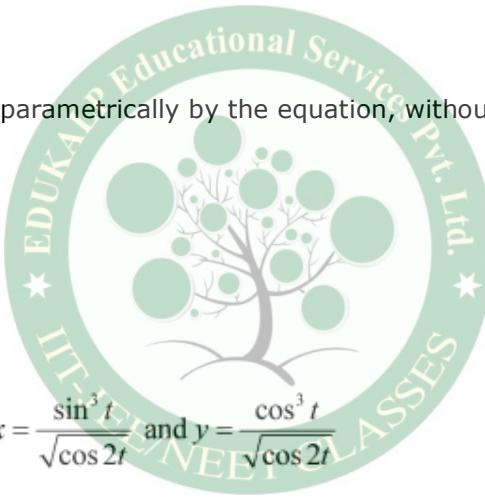
parameter, find  $\frac{dy}{dx}$ .

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

Answer

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}} \text{ and } y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

The given equations are



$$\begin{aligned} \text{Then, } \frac{dx}{dt} &= \frac{d}{dt} \left[ \frac{\sin^3 t}{\sqrt{\cos 2t}} \right] \\ &= \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\sin^3 t) - \sin^3 t \cdot \frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t} \\ &= \frac{\sqrt{\cos 2t} \cdot 3\sin^2 t \cdot \frac{d}{dt}(\sin t) - \sin^3 t \times \frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}(\cos 2t)}{\cos 2t} \\ &= \frac{3\sqrt{\cos 2t} \cdot \sin^2 t \cos t - \frac{\sin^3 t}{2\sqrt{\cos 2t}} \cdot (-2\sin 2t)}{\cos 2t} \\ &= \frac{3\cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t}{\cos 2t \sqrt{\cos 2t}} \end{aligned}$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{d}{dt} \left[ \frac{\cos^3 t}{\sqrt{\cos 2t}} \right] \\ &= \frac{\sqrt{\cos 2t} \cdot \frac{d}{dt}(\cos^3 t) - \cos^3 t \cdot \frac{d}{dt}(\sqrt{\cos 2t})}{\cos 2t} \\ &= \frac{\sqrt{\cos 2t} \cdot 3\cos^2 t \cdot \frac{d}{dt}(\cos t) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} \cdot \frac{d}{dt}(\cos 2t)}{\cos 2t} \\ &= \frac{3\sqrt{\cos 2t} \cdot \cos^2 t (-\sin t) - \cos^3 t \cdot \frac{1}{2\sqrt{\cos 2t}} \cdot (-2\sin 2t)}{\cos 2t} \\ &= \frac{-3\cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t \sin 2t}{\cos 2t \cdot \sqrt{\cos 2t}} \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{dy}{dx} &= \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-3\cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t \sin 2t}{3\cos 2t \sin^2 t \cos t + \sin^3 t \sin 2t} \\
 &= \frac{-3\cos 2t \cdot \cos^2 t \cdot \sin t + \cos^3 t (2\sin t \cos t)}{3\cos 2t \sin^2 t \cos t + \sin^3 t (2\sin t \cos t)} \\
 &= \frac{\sin t \cos t [-3\cos 2t \cdot \cos t + 2\cos^3 t]}{\sin t \cos t [3\cos 2t \sin t + 2\sin^3 t]} \\
 &= \frac{[-3(2\cos^2 t - 1)\cos t + 2\cos^3 t]}{[3(1 - 2\sin^2 t)\sin t + 2\sin^3 t]} \quad \left[ \begin{array}{l} \cos 2t = (2\cos^2 t - 1), \\ \cos 2t = (1 - 2\sin^2 t) \end{array} \right] \\
 &= \frac{-4\cos^3 t + 3\cos t}{3\sin t - 4\sin^3 t} \\
 &= \frac{-\cos 3t}{\sin 3t} \\
 &= -\cot 3t \quad \left[ \begin{array}{l} \cos 3t = 4\cos^3 t - 3\cos t, \\ \sin 3t = 3\sin t - 4\sin^3 t \end{array} \right]
 \end{aligned}$$

**Question 8:**

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = a \left( \cos t + \log \tan \frac{t}{2} \right), \quad y = a \sin t$$

Answer

$$x = a \left( \cos t + \log \tan \frac{t}{2} \right) \text{ and } y = a \sin t$$

The given equations are

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$$\begin{aligned}
 \text{Then, } \frac{dx}{dt} &= a \cdot \left[ \frac{d}{dt}(\cos t) + \frac{d}{dt} \left( \log \tan \frac{t}{2} \right) \right] \\
 &= a \left[ -\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \frac{d}{dt} \left( \tan \frac{t}{2} \right) \right] \\
 &= a \left[ -\sin t + \cot \frac{t}{2} \cdot \sec^2 \frac{t}{2} \cdot \frac{d}{dt} \left( \frac{t}{2} \right) \right] \\
 &= a \left[ -\sin t + \frac{\cos \frac{t}{2}}{\sin \frac{t}{2}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2} \right] \\
 &= a \left[ -\sin t + \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}} \right] \\
 &= a \left( -\sin t + \frac{1}{\sin t} \right) \\
 &= a \left( \frac{-\sin^2 t + 1}{\sin t} \right) \\
 &= a \frac{\cos^2 t}{\sin t}
 \end{aligned}$$

$$\frac{dy}{dt} = a \frac{d}{dt}(\sin t) = a \cos t$$

$$\therefore \frac{dy}{dx} = \frac{\left( \frac{dy}{dt} \right)}{\left( \frac{dx}{dt} \right)} = \frac{a \cos t}{\left( a \frac{\cos^2 t}{\sin t} \right)} = \frac{\sin t}{\cos t} = \tan t$$

#### **Question 9:**

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = a \sec \theta, y = b \tan \theta$$

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**Answer**

The given equations are  $x = a \sec \theta$  and  $y = b \tan \theta$

$$\text{Then, } \frac{dx}{d\theta} = a \cdot \frac{d}{d\theta}(\sec \theta) = a \sec \theta \tan \theta$$

$$\frac{dy}{d\theta} = b \cdot \frac{d}{d\theta}(\tan \theta) = b \sec^2 \theta$$

$$\therefore \frac{dy}{dx} = \frac{\left( \frac{dy}{d\theta} \right)}{\left( \frac{dx}{d\theta} \right)} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \sec \theta \cot \theta = \frac{b \cos \theta}{a \cos \theta \sin \theta} = \frac{b}{a} \times \frac{1}{\sin \theta} = \frac{b}{a} \operatorname{cosec} \theta$$

**Question 10:**

If  $x$  and  $y$  are connected parametrically by the equation, without eliminating the

parameter, find  $\frac{dy}{dx}$ .

$$x = a(\cos \theta + \theta \sin \theta), y = a(\sin \theta - \theta \cos \theta)$$

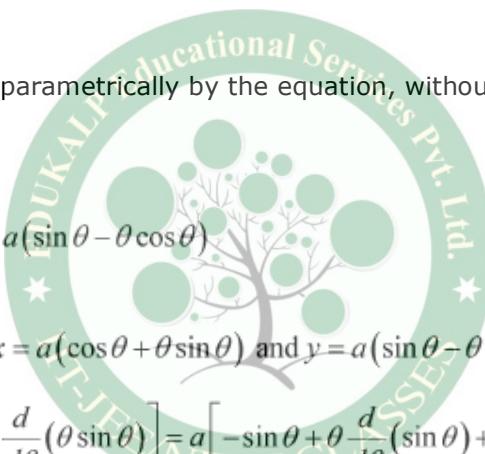
**Answer**

$$\text{The given equations are } x = a(\cos \theta + \theta \sin \theta) \text{ and } y = a(\sin \theta - \theta \cos \theta)$$

$$\begin{aligned} \text{Then, } \frac{dx}{d\theta} &= a \left[ \frac{d}{d\theta} \cos \theta + \frac{d}{d\theta} (\theta \sin \theta) \right] = a \left[ -\sin \theta + \theta \frac{d}{d\theta}(\sin \theta) + \sin \theta \frac{d}{d\theta}(\theta) \right] \\ &= a[-\sin \theta + \theta \cos \theta + \sin \theta] = a\theta \cos \theta \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\theta} &= a \left[ \frac{d}{d\theta}(\sin \theta) - \frac{d}{d\theta}(\theta \cos \theta) \right] = a \left[ \cos \theta - \left\{ \theta \frac{d}{d\theta}(\cos \theta) + \cos \theta \cdot \frac{d}{d\theta}(\theta) \right\} \right] \\ &= a[\cos \theta + \theta \sin \theta - \cos \theta] \\ &= a\theta \sin \theta \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{\left( \frac{dy}{d\theta} \right)}{\left( \frac{dx}{d\theta} \right)} = \frac{a\theta \sin \theta}{a\theta \cos \theta} = \tan \theta$$



**Question 11:**

If  $x = \sqrt{a^{\sin^{-1} t}}$ ,  $y = \sqrt{a^{\cos^{-1} t}}$ , show that  $\frac{dy}{dx} = -\frac{y}{x}$

**Answer**

The given equations are  $x = \sqrt{a^{\sin^{-1} t}}$  and  $y = \sqrt{a^{\cos^{-1} t}}$

$$x = \sqrt{a^{\sin^{-1} t}} \text{ and } y = \sqrt{a^{\cos^{-1} t}}$$

$$\Rightarrow x = (a^{\sin^{-1} t})^{\frac{1}{2}} \text{ and } y = (a^{\cos^{-1} t})^{\frac{1}{2}}$$

$$\Rightarrow x = a^{\frac{1}{2}\sin^{-1} t} \text{ and } y = a^{\frac{1}{2}\cos^{-1} t}$$

$$\text{Consider } x = a^{\frac{1}{2}\sin^{-1} t}$$

Taking logarithm on both the sides, we obtain

$$\log x = \frac{1}{2}\sin^{-1} t \log a$$

$$\therefore \frac{1}{x} \cdot \frac{dx}{dt} = \frac{1}{2} \log a \cdot \frac{d}{dt}(\sin^{-1} t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{x}{2} \log a \cdot \frac{1}{\sqrt{1-t^2}}$$

$$\Rightarrow \frac{dx}{dt} = \frac{x \log a}{2\sqrt{1-t^2}}$$

$$\text{Then, consider } y = a^{\frac{1}{2}\cos^{-1} t}$$

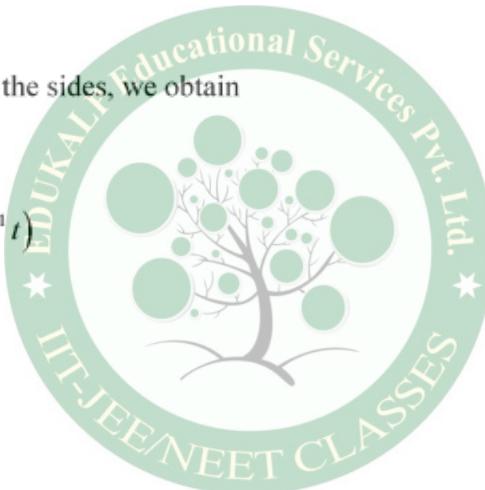
Taking logarithm on both the sides, we obtain

$$\log y = \frac{1}{2}\cos^{-1} t \log a$$

$$\therefore \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2} \log a \cdot \frac{d}{dt}(\cos^{-1} t)$$

$$\Rightarrow \frac{dy}{dt} = \frac{y \log a}{2} \cdot \left( \frac{-1}{\sqrt{1-t^2}} \right)$$

$$\Rightarrow \frac{dy}{dt} = \frac{-y \log a}{2\sqrt{1-t^2}}$$



$$\therefore \frac{dy}{dx} = \frac{\left( \frac{dy}{dt} \right)}{\left( \frac{dx}{dt} \right)} = \frac{\left( \frac{-y \log a}{2\sqrt{1-t^2}} \right)}{\left( \frac{x \log a}{2\sqrt{1-t^2}} \right)} = -\frac{y}{x}.$$

Hence, proved.

