Exercise 7.1

Question 1:

Find an anti-derivative (or integral) of the following functions by the method of inspection. sin 2x

Answer 1:

The anti-derivative of sin 2x is a function of x whose derivative is sin 2x.It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\Rightarrow \sin 2x = -\frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti-derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

Question 2:

Find an anti-derivative (or integral) of the following functions by the method of inspection. cos 3x

Answer 2:

The anti-derivative of cos 3x is a function of x whose derivative is cos 3x.

It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx}\left(\frac{1}{3}\sin 3x\right)$$

Therefore, the anti-derivative of $\cos 3x$ is $\frac{1}{3}\sin 3x$

Question 3:

Find an anti-derivative (or integral) of the following functions by the method of inspection. e^{2x}

Answer 3:

The anti-derivative of e^{2x} is the function of x whose derivative is e^{2x}.

It is known that,

$$\frac{d}{dx}\left(e^{2x}\right)=2e^{2x}$$

$$\Rightarrow e^{2x}=\frac{1}{2}\frac{d}{dx}\left(e^{2x}\right)$$

$$\therefore e^{2x}=\frac{d}{dx}\left(\frac{1}{2}e^{2x}\right)$$

Therefore, the anti-derivative of e^{2x} is $\frac{1}{2}e^{2x}$

Question 4:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $(ax + b)^2$

Answer 4:

The anti-derivative of $(ax + b)^2$ is the function of x whose derivative is $(ax + b)^2$.

It is known that,

$$\frac{d}{dx}(ax+b)^3=3a(ax+b)^2$$

$$\Rightarrow (ax+b)^2=\frac{1}{3a}\frac{d}{dx}(ax+b)^3$$

$$\therefore (ax+b)^2=\frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$

Question 5:

Find an anti-derivative (or integral) of the following functions by the method of inspection. $\sin 2x - 4e^{3x}$

Answer 5:

The anti-derivative of $\sin 2x - 4e^{3x}$ is the function of x whose derivative is $\sin 2x - 4e^{3x}$

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right) = \sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x - 4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}e^{3x}\right)$.

Question 6: $\int\left(4e^{3x}+1\right)dx$

Answer 6:

$$\int\left(4e^{3x}+1\right)dx$$

$$= 4\int e^{3x}dx + \int 1dx$$

$$= 4\left(\frac{e^{3x}}{3}\right) + x + C$$

$$= \frac{4}{3}e^{3x} + x + C$$

Question 7: $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Answer 7:

$\int x^2\left(1-\frac{1}{x^2}\right)dx$

$=\int\left(x^2-1\right)dx$

$=\int x^2dx-\int 1dx$

$=\frac{x^3}{3}-x+C$

Question 8: $\int\left(ax^2+bx+c\right)dx$

Answer 8:

$\int\left(ax^2+bx+c\right)dx$

$=a\int x^2dx+b\int xdx+c\int 1.dx$

$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$

$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$

Question 9: $\int\left(2x^2+e^x\right)dx$

Answer 9:

$\int\left(2x^2+e^x\right)dx$

$=2\int x^2dx+\int e^xdx$

$=2\left(\frac{x^3}{3}\right)+e^x+C$

$=\frac{2}{3}x^3+e^x+C$

Question 10: $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$

Answer 10:

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx$$

$$=\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int x dx+\int\frac{1}{x}dx-2\int 1.dx$$

$$=\frac{x^2}{2}+\log|x|-2x+C$$

Question 11: $\int\frac{x^3+5x^2-4}{x^2}dx$

Answer 11:

$$\int\frac{x^3+5x^2-4}{x^2}dx$$

$$=\int\left(x+5-4x^{-2}\right)dx$$

$$=\int x dx+5\int 1.dx-4\int x^{-2}dx$$

$$=\frac{x^2}{2}+5x-4\left(\frac{x^{-1}}{-1}\right)+C$$

$$=\frac{x^2}{2}+5x+\frac{4}{x}+C$$

Question 12: $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

Answer 12:

$$\int \frac{x^3+3x+4}{\sqrt{x}}dx$$

$$= \int\left(x^{\frac{5}{2}}+3x^{\frac{1}{2}}+4x^{-\frac{1}{2}}\right)dx$$

$$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}}+\frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8x^{\frac{1}{2}}+C$$

$$= \frac{2}{7}x^{\frac{7}{2}}+2x^{\frac{3}{2}}+8\sqrt{x}+C$$

Question 13: $\int \frac{x^3-x^2+x-1}{x-1}dx$

Answer 13:

$$\int \frac{x^3-x^2+x-1}{x-1}dx$$

On dividing, we obtain

$$= \int\left(x^2+1\right)dx$$

$$= \int x^2 dx + \int 1 dx$$

$$= \frac{x^3}{3}+x+C$$

Question 14: $\int (1-x)\sqrt{x}dx$

Answer 14:

$$\int (1-x)\sqrt{x}dx$$

$$= \int\left(\sqrt{x} - x^{\frac{3}{2}}\right)dx$$

$$= \int x^{\frac{1}{2}}dx - \int x^{\frac{3}{2}}dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$$

Question 15: $\int \sqrt{x}\left(3x^2+2x+3\right)dx$

Answer 15:

$$\int \sqrt{x}\left(3x^2+2x+3\right)dx$$

$$= \int\left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}}\right)dx$$

$$= 3\int x^{\frac{5}{2}}dx + 2\int x^{\frac{3}{2}}dx + 3\int x^{\frac{1}{2}}dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}}\right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + 3\frac{\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + C$$

$$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$