Exercise 7.10

Question 1:

$$\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$$

Answer 1:

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \sin^2 x dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^2 x\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

Question 2:

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Answer 2:

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos}}{\sqrt{\cos} + \sqrt{\sin x}} dx \qquad \ldots(2)$$

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Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

Question 3:

$$\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x\,dx}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}$$

Answer 3:

$$\text{Let } I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \quad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)+\cos^{\frac{3}{2}}\left(\frac{\pi}{2}-x\right)}dx \quad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}}1\cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

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Question 4:

$$\int_0^{\frac{\pi}{2}} \frac{\cos^5 x dx}{\sin^5 x+\cos^5 x}$$

Answer 4:

$$\text{Let } I=\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sin^5 x+\cos^5 x} dx \quad \ldots(1)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)} dx \quad \left(\int_0^a f(x) dx=\int_0^a f(a-x) dx\right)$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\sin^5 x+\cos^5 x} dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}} \frac{\sin^5 x+\cos^5 x}{\sin^5 x+\cos^5 x} dx$$

$$\Rightarrow 2I=\int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I=[x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I=\frac{\pi}{2}$$

$$\Rightarrow I=\frac{\pi}{4}$$

Question 5:

$$\int_{-5}^{5}|x+2| dx$$

Answer 5:

$$\text{Let } I=\int_{-5}^{5}|x+2| dx$$

It can be seen that (x + 2) ≤ 0 on [−5, −2] and (x + 2) ≥ 0 on [−2, 5].

$$\therefore I=\int_{-5}^{-2}-(x+2)dx+\int_{-2}^{5}(x+2)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$I=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$

$$=-\left[\frac{(-2)^2}{2}+2(-2)-\frac{(-5)^2}{2}-2(-5)\right]+\left[\frac{(5)^2}{2}+2(5)-\frac{(-2)^2}{2}-2(-2)\right]$$

$$=-\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$$

$$=-2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$$

$$=29$$

Question 6:

$$\int_2^8|x-5|\,dx$$

Answer 6:

Let $I=\int_2^8|x-5|\,dx$

It can be seen that (x − 5) ≤ 0 on [2, 5] and (x − 5) ≥ 0 on [5, 8].

$$I=\int_2^5-(x-5)dx+\int_2^8(x-5)dx \qquad \left(\int_a^b f(x)=\int_a^c f(x)+\int_c^b f(x)\right)$$

$$=-\left[\frac{x^2}{2}-5x\right]_2^5+\left[\frac{x^2}{2}-5x\right]_5^8$$

$$=-\left[\frac{25}{2}-25-2+10\right]+\left[32-40-\frac{25}{2}+25\right]$$

$$=9$$

Question 7:

$$\int_0^1 x(1-x)^n\,dx$$

Answer 7:

Let $I = \int_0^1 x(1-x)^n\,dx$

$$\therefore I = \int_0^1 (1-x)(1-(1-x))^n\,dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$= \int_0^1 (1-x)(x)^n\,dx$$

$$= \int_0^1 \left(x^n - x^{n+1}\right)dx$$

$$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1$$

$$= \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{(n+2)-(n+1)}{(n+1)(n+2)}$$

$$= \frac{1}{(n+1)(n+2)}$$

Question 8:

$$\int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$$

Answer 8:

Let $I = \int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx \qquad \ldots(1)$

$$\therefore I = \int_0^{\frac{\pi}{4}} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan x}{1+\tan}\right\}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log\frac{2}{(1+\tan x)}dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2\,dx - \int_0^{\frac{\pi}{4}} \log\,(1+\tan x)dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \log 2dx - I \qquad [\text{From } (1)]$$

$$\Rightarrow 2I = [x\log 2]_0^{\frac{\pi}{4}}$$

$$\Rightarrow 2I = \frac{\pi}{4}\log 2$$

$$\Rightarrow I = \frac{\pi}{8}\log 2$$

Question 9:

$$\int_0^2 x\sqrt{2-x}\,dx$$

Answer 9:

$$\text{Let } I = \int_0^2 x\sqrt{2-x}\,dx$$

$$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\}dx$$

$$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$

$$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4\times 2\sqrt{2}}{3} - \frac{2}{5}\times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

Question 10:

$$\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$$

Answer 10:

$$\text{Let } I=\int_0^{\frac{\pi}{2}}(2\log\sin x-\log\sin 2x)dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log(2\sin x\cos x)\}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{2\log\sin x-\log\sin x-\log\cos x-\log 2\}dx$$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\sin x-\log\cos x-\log 2\}dx \quad ...(1)$$

It is known that, $\left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\{\log\cos x-\log\sin x-\log 2\}dx \quad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^{\frac{\pi}{2}}(-\log 2-\log 2)dx$$

$$\Rightarrow 2I=-2\log 2\int_0^{\frac{\pi}{2}}1\,dx$$

$$\Rightarrow I=-\log 2\left[\frac{\pi}{2}\right]$$

$$\Rightarrow I=\frac{\pi}{2}(-\log 2)$$

$$\Rightarrow I=\frac{\pi}{2}\left[\log\frac{1}{2}\right]$$

$$\Rightarrow I=\frac{\pi}{2}\log\frac{1}{2}$$

Question 11:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\,dx$$

Answer 11:

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\,dx$$

As $\sin^2(-x) = (\sin(-x))^2 = (-\sin x)^2 = \sin^2 x$, therefore, $\sin^2 x$ is an even function.

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

$$I = 2\int_0^{\frac{\pi}{2}} \sin^2 x\,dx$$

$$= 2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}\,dx$$

$$= \int_0^{\frac{\pi}{2}} (1-\cos 2x)\,dx$$

$$= \left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

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Question 12:

$$\int_0^{\pi} \frac{x\,dx}{1+\sin x}$$

Answer 12:

$$\text{Let } I = \int_0^{\pi} \frac{x\,dx}{1+\sin x} \quad \ldots(1)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{1+\sin x}\,dx \quad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\pi} \frac{\pi}{1+\sin x} dx$$
$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)} dx$$
$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{1-\sin x}{\cos^2 x} dx$$
$$\Rightarrow 2I = \pi \int_0^{\pi} \{\sec^2 x - \tan x \sec x\} dx$$
$$\Rightarrow 2I = \pi [\tan x - \sec x]_0^{\pi}$$
$$\Rightarrow 2I = \pi [2]$$
$$\Rightarrow I = \pi$$

Question 13:

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$$

Answer 13:

$$\text{Let } I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx \qquad ...(1)$$

As $\sin^7(-x) = (\sin(-x))^7 = (-\sin x)^7 = -\sin^7 x$, therefore, $\sin^2 x$ is an odd function.

It is known that, if f(x) is an odd function, then $\int_{-a}^{a} f(x) dx = 0$

$$\therefore I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0$$

Question 14:

$$\int_0^{2\pi} \cos^5 x dx$$

Answer 14:

$$\text{Let } I = \int_0^{2\pi} \cos^5 x dx \qquad ...(1)$$

$$\cos^5(2\pi - x) = \cos^5 x$$

It is known that,

$$\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{ if } f(2a-x) = f(x)$$
$$= 0 \text{ if } f(2a-x) = -f(x)$$

$$\therefore I = 2\int_0^{\pi} \cos^5 x dx$$

$$\Rightarrow I = 2(0) = 0 \qquad \left[\cos^5(\pi - x) = -\cos^5 x\right]$$

Question 15:

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$$

Answer 15:

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2} - x\right) - \cos\left(\frac{\pi}{2} - x\right)}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{0}{1 + \sin x \cos x} dx$$

$$\Rightarrow I = 0$$

Question 16:

$$\int_0^{\pi} \log(1 + \cos x) dx$$

Answer 16:

$$\text{Let } I = \int_0^{\pi} \log(1 + \cos x) dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 + \cos(\pi - x)) dx \qquad \left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$$

$$\Rightarrow I = \int_0^{\pi} \log(1 - \cos x) dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^\pi \{\log(1+\cos x) + \log(1-\cos x)\}\,dx$$

$$\Rightarrow 2I = \int_0^\pi \log(1-\cos^2 x)\,dx$$

$$\Rightarrow 2I = \int_0^\pi \log\sin^2 x\,dx$$

$$\Rightarrow 2I = 2\int_0^\pi \log\sin x\,dx$$

$$\Rightarrow I = \int_0^\pi \log\sin x\,dx \qquad ...(3)$$

sin (π − x) = sin x

$$\therefore I = 2\int_0^{\frac{\pi}{2}} \log\sin x\,dx \qquad ...(4)$$

$$\Rightarrow I = 2\int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2}-x\right)dx = 2\int_0^{\frac{\pi}{2}} \log\cos x\,dx \qquad ...(5)$$

Adding (4) and (5), we obtain

$$2I = 2\int_0^{\frac{\pi}{2}} (\log\sin x + \log\cos x)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log\sin x + \log\cos x + \log 2 - \log 2)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} (\log 2\sin x\cos x - \log 2)\,dx$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \log\sin 2x\,dx - \int_0^{\frac{\pi}{2}} \log 2\,dx$$

Let 2x = t ⇒ 2dx = dt

When x = 0, t = 0

$$\therefore I = \frac{1}{2}\int_0^\pi \log\sin t\,dt - \frac{\pi}{2}\log 2$$

$$\Rightarrow I = \frac{1}{2}I - \frac{\pi}{2}\log 2$$

$$\Rightarrow \frac{I}{2} = -\frac{\pi}{2}\log 2$$

$$\Rightarrow I = -\pi\log 2$$

Question 17:

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

Answer 17:

$$\text{Let } I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx \qquad ...(1)$$

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I = \int_0^a 1\cdot dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

Question 18:

$$\int_0^4 |x-1|\,dx$$

Answer 18:

$$I = \int_0^4 |x-1|\,dx$$

It can be seen that, $(x - 1) \le 0$ when $0 \le x \le 1$ and $(x - 1) \ge 0$ when $1 \le x \le 4$

$$I = \int_0^1 |x-1|\,dx + \int_1^4 |x-1|\,dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1)\,dx + \int_1^4 (x-1)\,dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Question 19:

Show that $\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx,$ if f and g are defined as $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$

Answer 19:

$$\text{Let } I=\int_0^a f(x)g(x)dx \qquad ...(1)$$

$$\Rightarrow I=\int_0^a f(a-x)g(a-x)dx \qquad \left(\int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$\Rightarrow I=\int_0^a f(x)g(a-x)dx \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2I=\int_0^a \{f(x)g(x)+f(x)g(a-x)\}dx$$

$$\Rightarrow 2I=\int_0^a f(x)\{g(x)+g(a-x)\}dx$$

$$\Rightarrow 2I=\int_0^a f(x)\times 4\,dx \qquad [g(x)+g(a-x)=4]$$

$$\Rightarrow I=2\int_0^a f(x)dx$$

Question 20:

The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$ is

A. 0

B. 2

C. π

D. 1

Answer 20:

$$\text{Let } I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^3+x\cos x+\tan^5 x+1\right)dx$$

$$\Rightarrow I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^3dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos x+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\tan^5 x dx+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}1\cdot dx$$

It is known that if f(x) is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

if f(x) is an odd function, then $\int_{-a}^{a} f(x)dx = 0$

and

$$I = 0+0+0+2\int_{0}^{\frac{\pi}{2}} 1 \cdot dx$$

$$= 2[x]_{0}^{\frac{\pi}{2}}$$

$$= \frac{2\pi}{2}$$

$$= \pi$$

Hence, the correct Answer is C.

Question 21:

The value of $\int_{0}^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ is

A. 2

B. $\frac{3}{4}$

C. 0

D. -2

Answer 21:

$$\text{Let } I = \int_{0}^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx \quad ...(1)$$

$$\Rightarrow I = \int_{0}^{\frac{\pi}{2}} \log\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx \quad \left(\int_{0}^{a} f(x)dx = \int_{0}^{a} f(a-x)dx\right)$$

$$\Rightarrow I = \int_{0}^{\frac{\pi}{2}} \log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx \quad ...(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \left\{ \log\left(\frac{4+3\sin x}{4+3\cos x}\right) + \log\left(\frac{4+3\cos x}{4+3\sin x}\right) \right\} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right) dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \log 1 dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 0 dx$$

$$\Rightarrow I = 0$$

Hence, the correct Answer is C.

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