Exercise 7.5

Integrate the rational functions in Exercises 1 to 21.

Question 1:
$$\frac{x}{(x+1)(x+2)}$$

Answer 1:

Let
$$\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$
$$\Rightarrow x = A(x+2) + B(x+1)$$
 convertion

e obtain

Equating the coefficients of x and constant term, we obtain

$$A + B = 1$$

$$2A + B = 0$$
On solving, we obtain

$$A = -1 \text{ and } B = 2$$

$$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$$

$$\Rightarrow \int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)} dx$$

$$= -\log|x+1| + 2\log|x+2| + C$$

$$= \log(x+2)^2 - \log|x+1| + C$$

$$= \log \frac{\left(x+2\right)^2}{\left(x+1\right)} + C$$

Question 2: $\frac{1}{x^2-9}$ Answer 2: Let $\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$ 1 = A(x-3) + B(x+3)

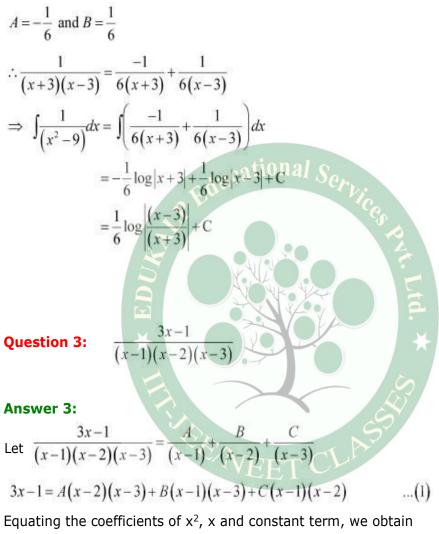
Equating the coefficients of x and constant term, we obtain A + B = 0



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-3A + 3B = 1

On solving, we obtain



A + B + C = 0- 5A - 4B - 3C = 3 6A + 3B + 2C = - 1 Solving these equations, we obtain A = 1, B = -5, and C = 4

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \right\} dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

Question 4:
$$\frac{x}{(x-1)(x-2)(x+3)}$$
 Cational Services
Answer 4:
Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$
 $x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$...(1)

Equating the coefficients of x^2 , x and constant term, we obtain

A + B + C = 0- 5A - 4B - 3C = 1 6A + 4B + 2C = 0

Solving these equations, we obtain VEFT

$$A = \frac{1}{2}, B = -2, \text{ and } C = \frac{3}{2}$$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)} \right\} dx$$

$$= \frac{1}{2} \log|x-1| - 2\log|x-2| + \frac{3}{2} \log|x-3| + C$$

Question 5:
$$\frac{2x}{x^2+3x+2}$$

Answer 5:

Let
$$\frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$$

 $2x = A(x+2) + B(x+1) \qquad ...(1)$

Equating the coefficients of x^2 , x and constant term, we obtain

$$A + B = 2$$

 $2A + B = 0$

Solving these equations, we obtain

A = -2 and B = 4

$$\therefore \frac{2x}{(x+1)(x+2)} = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\Rightarrow \int \frac{2x}{(x+1)(x+2)} dx = \int \left\{ \frac{4}{(x+2)} - \frac{2}{(x+2)} \right\}$$

$$4 \log |x+2| - 2 \log |x+1| + 0$$

Question 6:

Answer 6:

It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1 - x^2)$ by x(1 - 2x), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2} \left(\frac{2-x}{x(1-2x)} \right)$$

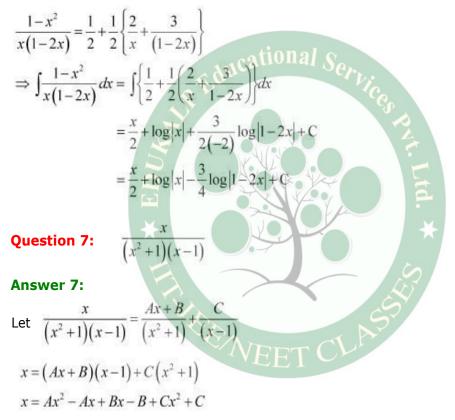
Let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$
 $\Rightarrow (2-x) = A(1-2x) + Bx$...(1)

Equating the coefficients of x^2 , x and constant term, we obtain

-2A + B = -1And A = 2Solving these equations, we obtain A = 2 and B = 3

 $\therefore \frac{2-x}{x(1-2x)} = \frac{2}{x} + \frac{3}{1-2x}$

Substituting in equation (1), we obtain



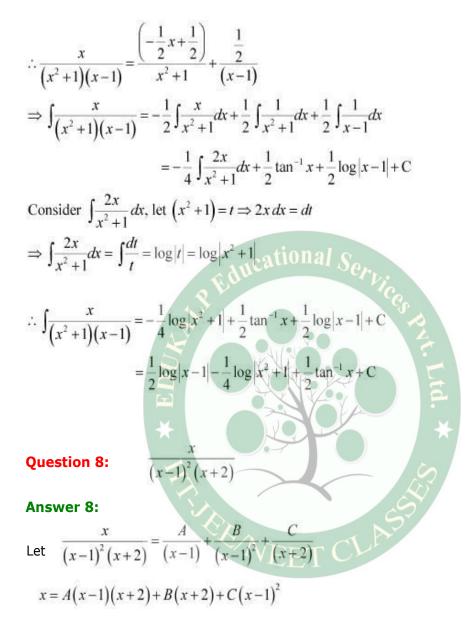
Equating the coefficients of x^2 , x, and constant term, we obtain

$$A + C = 0$$
$$-A + B = 1$$
$$-B + C = 0$$

On solving these equations, we obtain

$$A = -\frac{1}{2}, B = \frac{1}{2}, \text{ and } C = \frac{1}{2}$$

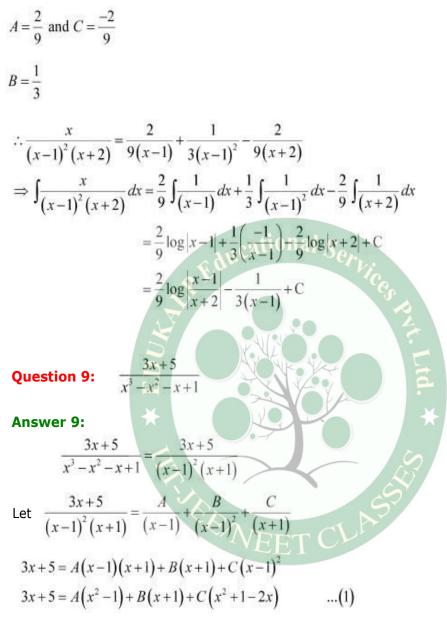
From equation (1), we obtain



Substituting x = 1, we obtain

Equating the coefficients of x^2 , x and constant term, we obtain

A + C = 0 A + B - 2C = 1 -2A + 2B + C = 0On solving, we obtain



Equating the coefficients of x^2 , x and constant term, we obtain

$$A + C = 0$$

$$B - 2C = 3$$

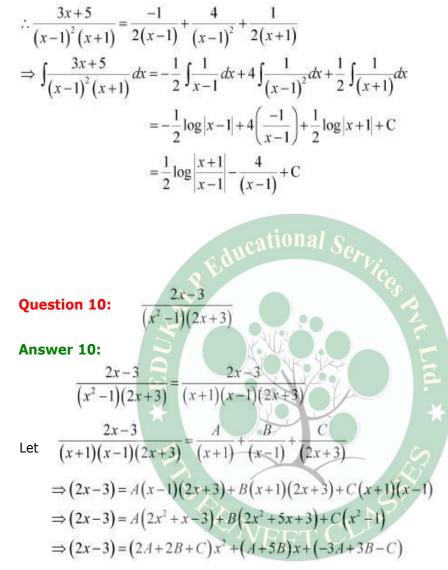
$$- A + B + C = 5$$

On solving, we obtain

$$B = 4$$

$$A = -\frac{1}{2} \text{ and } C = \frac{1}{2}$$

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Equating the coefficients of x^2 , x and constant, we obtain

2A + 2B + C = 0A + 5B = 2 - 3A + 3B - C = - 3

On solving, we obtain

$$B = -\frac{1}{10}, A = \frac{5}{2}, \text{ and } C = -\frac{24}{5}$$

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$$\therefore \frac{2x-3}{(x+1)(x-1)(2x+3)} = \frac{5}{2(x+1)} - \frac{1}{10(x-1)} - \frac{24}{5(2x+3)}$$

$$\Rightarrow \int \frac{2x-3}{(x^2-1)(2x+3)} dx = \frac{5}{2} \int \frac{1}{(x+1)} dx - \frac{1}{10} \int \frac{1}{x-1} dx - \frac{24}{55} \int \frac{1}{(2x+3)} dx$$

$$= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{24}{5\times 2} \log|2x+3|$$

$$= \frac{5}{2} \log|x+1| - \frac{1}{10} \log|x-1| - \frac{24}{5\times 2} \log|2x+3| + C$$
Question 11:

$$\frac{5x}{(x+1)(x^2-4)} = \frac{5x}{(x+1)(x+2)(x-2)}$$
Let
$$\frac{5x}{(x+1)(x+2)(x-2)} = \frac{4}{(x+1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)}$$
Summary the coefficients of x², x and constant, we obtain
A + B + C = 0
- B + 3C = 5 and
- 4A - 2B + 2C = 0
On solving, we obtain

$$A = \frac{5}{3}, B = -\frac{5}{2}, and C = \frac{5}{6}$$

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3} \int \frac{1}{(x+1)} dx - \frac{5}{2} \int \frac{1}{(x+2)} dx + \frac{5}{6} \int \frac{1}{(x-2)} dx$$

$$= \frac{5}{3} \log|x+1| - \frac{5}{2} \log|x+2| + \frac{5}{6} \log|x-2| + C$$

Question 12:
$$\frac{x^3 + x + 1}{x^2 - 1}$$

Answer 12:

It can be seen that the given integrand is not a proper fraction. Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we obtain

$$\frac{x^{3} + x + 1}{x^{2} - 1} = x + \frac{2x + 1}{x^{2} - 1}$$
Let $\frac{2x + 1}{x^{2} - 1} = \frac{A}{(x + 1)} + \frac{B}{(x - 1)}$ uncational Server
 $2x + 1 = A(x - 1) + B(x + 1)$...(1)
Equating the coefficients of x and constant, we obtain

Equating the coefficients of x and constant, we obtain

$$A + B = 2$$

$$-A + B = 1$$
On solving, we obtain

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

$$\therefore \frac{x^3 + x + 1}{x^2 - 1} = x + \frac{1}{2(x+1)} + \frac{3}{2(x-1)}$$

$$\Rightarrow \int \frac{x^3 + x + 1}{x^2 - 1} dx = \int x dx + \frac{1}{2} \int \frac{1}{(x+1)} dx + \frac{3}{2} \int \frac{1}{(x-1)} dx$$

$$= \frac{x^2}{2} + \frac{1}{2} \log|x+1| + \frac{3}{2} \log|x-1| + C$$

Question 13:
$$\frac{2}{(1-x)(1+x^2)}$$

Answer 13:

Let
$$\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$$

 $2 = A(1+x^2) + (Bx+C)(1-x)$
 $2 = A + Ax^2 + Bx - Bx^2 + C - Cxucational$

Equating the coefficient of x^2 , x, and constant term, we obtain

A - B = 0B - C = 0A + C = 2

On solving these equations, we obtain

A = 1, B = 1, and C = 1

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\int \frac{1}{x-1} dx + \frac{1}{2} \int \frac{2x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$$

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Question 14:
$$\frac{3x-1}{(x+2)^2}$$

Answer 14:

Let
$$\frac{3x-1}{(x+2)^2} = \frac{A}{(x+2)} + \frac{B}{(x+2)^2}$$

 $\Rightarrow 3x-1 = A(x+2) + B$

Equating the coefficient of x and constant term, we obtain

A = 3
2A + B = -1
$$\Rightarrow$$
 B = -7
 $\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} - \frac{7}{(x+2)^2}$ Example to the formula of th

Equating the coefficient of x^3 , x^2 , x, and constant term, we obtain

A + B + C = 0-A + B + D = 0A + B - C = 0

$$-A + B - D = 1$$

On solving these equations, we obtain

$$A = -\frac{1}{4}, B = \frac{1}{4}, C = 0, \text{ and } D = -\frac{1}{2}$$

$$\therefore \frac{1}{x^4 - 1} = \frac{-1}{4(x+1)} + \frac{1}{4(x-1)} - \frac{1}{2(x^2+1)}$$

$$\Rightarrow \int \frac{1}{x^4 - 1} dx = -\frac{1}{4} \log|x-1| + \frac{1}{4} \log|x-1| - \frac{1}{2} \tan^{-1} x + C$$

$$= \frac{1}{4} \log \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \tan^{-1} x + C$$

 $x(x^{n}+1)$

Question 16:

[Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Answer 16:

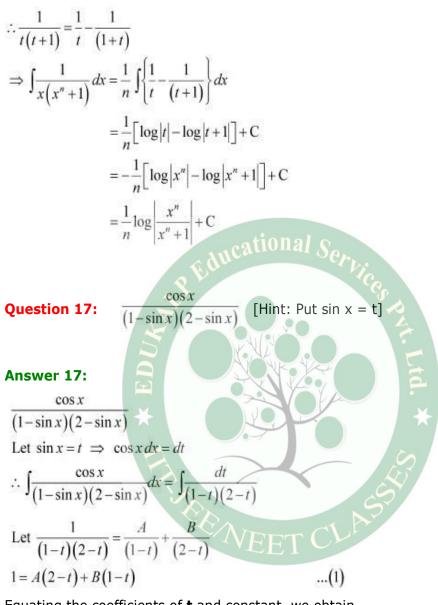
$$\frac{1}{x(x''+1)}$$

Multiplying numerator and denominator by x^{n-1} , we obtain

$$\frac{1}{x(x^{n}+1)} = \frac{x^{n-1}}{x^{n-1}x(x^{n}+1)} = \frac{x^{n-1}}{x^{n}(x^{n}+1)}$$

Let $x^{n} = t \implies x^{n-1}dx = dt$
 $\therefore \int \frac{1}{x(x^{n}+1)}dx = \int \frac{x^{n-1}}{x^{n}(x^{n}+1)}dx = \frac{1}{n}\int \frac{1}{t(t+1)}dt$
Let $\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{(t+1)}$
 $1 = A(1+t) + Bt$...(1)

Equating the coefficients of t and constant, we obtain A = 1 and B = -1

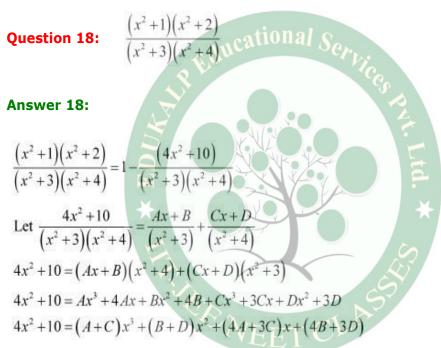


Equating the coefficients of **t** and constant, we obtain -2A - B = 0 and 2A + B = 1

On solving, we obtain A = 1 and B = -1

$$\therefore \frac{1}{(1-t)(2-t)} = \frac{1}{(1-t)} - \frac{1}{(2-t)}$$

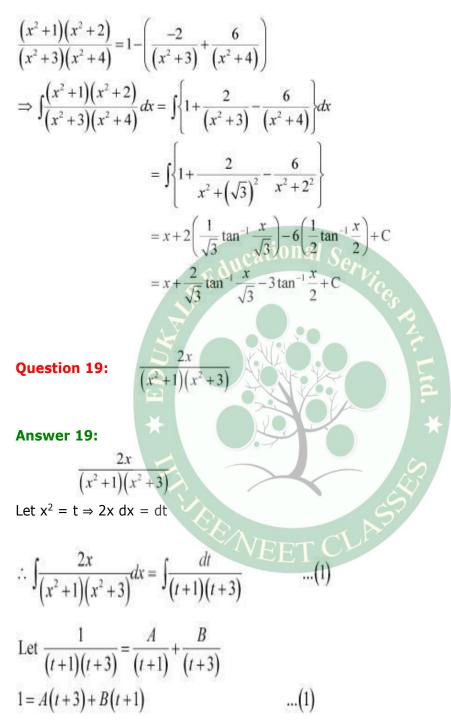
$$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)} dx = \int \left\{ \frac{1}{1-t} - \frac{1}{(2-t)} \right\} dt$$
$$= -\log|1-t| + \log|2-t| + C$$
$$= \log\left|\frac{2-t}{1-t}\right| + C$$
$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$



Equating the coefficients of x^3 , x^2 , x, and constant term, we obtain

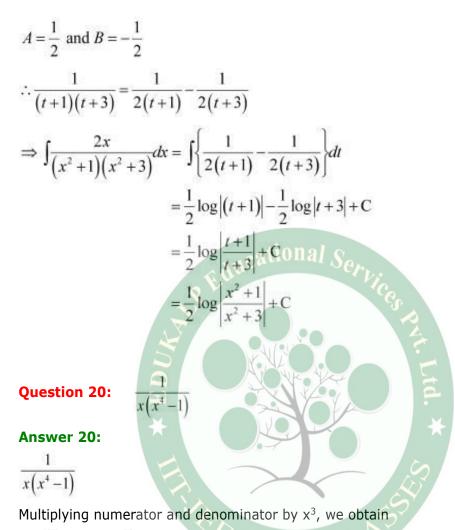
A + C = 0 B + D = 4 4A + 3C = 0 4B + 3D = 10On solving these equations, we obtain A = 0, B = -2, C = 0, and D = 6

$$\therefore \frac{4x^2 + 10}{\left(x^2 + 3\right)\left(x^2 + 4\right)} = \frac{-2}{\left(x^2 + 3\right)} + \frac{6}{\left(x^2 + 4\right)}$$



Equating the coefficients of t and constant, we obtain A + B = 0 and 3A + B = 1

On solving, we obtain



$$\frac{1}{x(x^4-1)} = \frac{x^3}{x^4(x^4-1)}$$

$$\therefore \int \frac{1}{x(x^4-1)} dx = \int \frac{x^3}{x^4(x^4-1)} dx$$

Let
$$x^4 = t \Rightarrow 4x^3dx = dt$$

$$\therefore \int \frac{1}{x(x^4 - 1)} dx = \frac{1}{4} \int \frac{dt}{t(t - 1)}$$
Let $\frac{1}{t(t - 1)} = \frac{A}{t} + \frac{B}{(t - 1)}$
 $1 = A(t - 1) + Bt$...(1)



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Equating the coefficients of t and constant, we obtain

A = -1 and B = 1

$$\Rightarrow \frac{1}{t(t+1)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{x(x^4-1)} dx = \frac{1}{4} \int \left\{ \frac{-1}{t} + \frac{1}{t-1} \right\} dt$$

$$= \frac{1}{4} \log \left| \frac{t-1}{t} \right| + C$$

$$= \frac{1}{4} \log \left| \frac{t-1}{t} \right| + C$$

$$= \frac{1}{4} \log \left| \frac{x^4 + 1}{x^4} \right| + C$$
Question 21:
Answer 21:

$$\frac{1}{(e^x - 1)}$$
Let $e^x = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int \frac{1}{e^x - 1} dx = \int \frac{1}{t-1} \times \frac{dt}{t} = \int \frac{1}{t(t-1)} dt$$
Let $\frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1}$

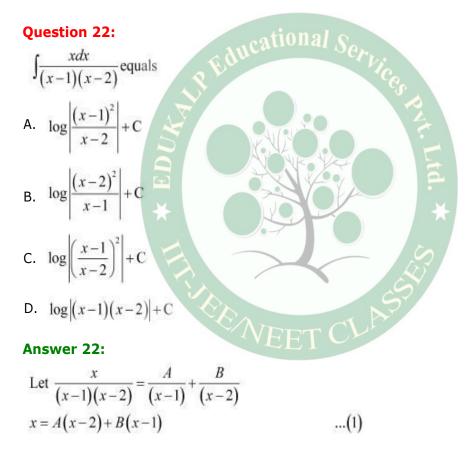
$$1 = A(t-1) + Bt$$
Equating the coefficients of t and constant, we obtain

A = -1 and B = 1

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$$\therefore \frac{1}{t(t-1)} = \frac{-1}{t} + \frac{1}{t-1}$$
$$\Rightarrow \int \frac{1}{t(t-1)} dt = \log \left| \frac{t-1}{t} \right| + C$$
$$= \log \left| \frac{e^x - 1}{e^x} \right| + C$$



Equating the coefficients of $\ x$ and constant, we obtain A = -1 and B = 2

$$\therefore \frac{x}{(x-1)(x-2)} = -\frac{1}{(x-1)} + \frac{2}{(x-2)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)} dx = \int \left\{ \frac{-1}{(x-1)} + \frac{2}{(x-2)} \right\} dx$$

$$= -\log|x-1| + 2\log|x-2| + C$$

$$= \log\left| \frac{(x-2)^2}{x-1} \right| + C$$

Hence, the correct Answer is B.

Question 23:

$$\int \frac{dx}{x(x^2+1)}$$
 equals

A.
$$\log |x| - \frac{1}{2} \log (x^2 + 1) + 0$$

B.
$$\log|x| + \frac{1}{2}\log(x^2 + 1) + C$$

C.
$$-\log|x| + \frac{1}{2}\log(x^2 + 1) + \frac{1}{2}\log(x^2 + 1)$$

D.
$$\frac{1}{2}\log|x| + \log(x^2 + 1) + C$$

Answer 23:

Let
$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

 $1 = A(x^2+1) + (Bx+C)x$

Equating the coefficients of x^2 , x, and constant term, we obtain



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On solving these equations, we obtain

A = 1, B = -1, and C = 0

$$\therefore \frac{1}{x(x^2+1)} = \frac{1}{x} + \frac{-x}{x^2+1}$$
$$\Rightarrow \int \frac{1}{x(x^2+1)} dx = \int \left\{ \frac{1}{x} - \frac{x}{x^2+1} \right\} dx$$
$$= \log|x| - \frac{1}{2} \log|x^2+1| + C^{10}$$

Hence, the correct Answer is A.

