Exercise 7.6

Integrate the functions in Exercises 1 to 22.

Question 1: x sin x

Answer 1:

Let I = $\int x \sin x \, dx$

Taking x as first function and sin x as second function and integrating by parts, we obtain

$$I = x\int \sin x \, dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin x \, dx\right\}dx$$

$$= x(-\cos x) - \int 1\cdot(-\cos x)\,dx$$

$$= -x\cos x + \sin x + C$$

Question 2: $x \sin 3x$

Answer 2:

Let I = $\int x \sin 3x \, dx$

Taking x as first function and sin 3x as second function and integrating by parts, we obtain

$$I = x\int \sin 3x \, dx - \int\left\{\left(\frac{d}{dx}x\right)\int \sin 3x \, dx\right\}$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1\cdot\left(\frac{-\cos 3x}{3}\right)dx$$

$$= \frac{-x\cos 3x}{3} + \frac{1}{3}\int \cos 3x \, dx$$

$$= \frac{-x\cos 3x}{3} + \frac{1}{9}\sin 3x + C$$

Question 3: $x^2 e^x$

Answer 3:

Let $I = \int x^2 e^x dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2 \int e^x dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x dx \right\} dx$$

$$= x^2 e^x - \int 2x \cdot e^x dx$$

$$= x^2 e^x - 2 \int x \cdot e^x dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2 \left[x \cdot \int e^x dx - \int \left\{ \left(\frac{d}{dx} x \right) \cdot \int e^x dx \right\} dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - e^x \right]$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$= e^x \left(x^2 - 2x + 2 \right) + C$$

Question 4: x logx

Answer 4:

Let $I = \int x \log x dx$

Taking log x as first function and x as second function and integrating by parts, we obtain

$$I = \log x \int x\, dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x\, dx \right\} dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

Question 5: x log 2x

Answer 5:

Let $I = \int x \log 2x dx$

Taking log 2x as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x\, dx - \int \left\{ \left(\frac{d}{dx} 2\log x \right) \int x\, dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

Question 6: x^2 log x

Answer 6:

Let $I = \int x^2 \log x\, dx$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I=\log x\int x^2\,dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int x^2dx\right\}dx$$

$$=\log x\left(\frac{x^3}{3}\right)-\int\frac{1}{x}\cdot\frac{x^3}{3}dx$$

$$=\frac{x^3\log x}{3}-\int\frac{x^2}{3}dx$$

$$=\frac{x^3\log x}{3}-\frac{x^3}{9}+C$$

Question 7: $x\sin^{-1}x$

Answer 7:

Let $I=\int x\sin^{-1}x\,dx$

Taking $\sin^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I=\sin^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int x\,dx\right\}dx$$

$$=\sin^{-1}x\left(\frac{x^2}{2}\right)-\int\frac{1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$=\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\int\left\{\frac{1-x^2}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$=\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\int\left\{\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right\}dx$$

$$=\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\left\{\int\sqrt{1-x^2}dx-\int\frac{1}{\sqrt{1-x^2}}dx\right\}$$

$$=\frac{x^2\sin^{-1}x}{2}+\frac{1}{2}\left\{\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x-\sin^{-1}x\right\}+C$$

$$=\frac{x^2\sin^{-1}x}{2}+\frac{x}{4}\sqrt{1-x^2}+\frac{1}{4}\sin^{-1}x-\frac{1}{2}\sin^{-1}x+C$$

$$=\frac{1}{4}\left(2x^2-1\right)\sin^{-1}x+\frac{x}{4}\sqrt{1-x^2}+C$$

Question 8: $x\tan^{-1}x$

Answer 8:

Let $I=\int x\tan^{-1}x\,dx$

Taking $\tan^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I=\tan^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int x\,dx\right\}dx$$

$$=\tan^{-1}x\left(\frac{x^2}{2}\right)-\int\frac{1}{1+x^2}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\left(\frac{x^2+1}{1+x^2}-\frac{1}{1+x^2}\right)dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$$

$$=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\left(x-\tan^{-1}x\right)+C$$

$$=\frac{x^2}{2}\tan^{-1}x-\frac{x}{2}+\frac{1}{2}\tan^{-1}x+C$$

Question 9: x $\cos^{-1}$x

Answer 9:

Let $I=\int x\cos^{-1}x\,dx$

Taking $\cos^{-1}x$ as first function and x as second function and integrating by parts, we obtain

$$I=\cos^{-1}x\int x\,dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int x\,dx\right\}dx$$

$$=\cos^{-1}x\frac{x^2}{2}-\int\frac{-1}{\sqrt{1-x^2}}\cdot\frac{x^2}{2}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\left\{\sqrt{1-x^2}+\left(\frac{-1}{\sqrt{1-x^2}}\right)\right\}dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}\int\sqrt{1-x^2}\,dx-\frac{1}{2}\int\left(\frac{-1}{\sqrt{1-x^2}}\right)dx$$

$$=\frac{x^2\cos^{-1}x}{2}-\frac{1}{2}I_1-\frac{1}{2}\cos^{-1}x \qquad ...(1)$$

where, $I_1=\int\sqrt{1-x^2}\,dx$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{d}{dx}\sqrt{1-x^2}\int x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-2x}{2\sqrt{1-x^2}}.x\,dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{-x^2}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\int\frac{1-x^2-1}{\sqrt{1-x^2}}dx$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{\int\sqrt{1-x^2}\,dx+\int\frac{-dx}{\sqrt{1-x^2}}\right\}$$

$$\Rightarrow I_1=x\sqrt{1-x^2}-\left\{I_1+\cos^{-1}x\right\}$$

$$\Rightarrow 2I_1=x\sqrt{1-x^2}-\cos^{-1}x$$

$$\therefore I_1=\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x$$

Substituting in (1), we obtain

$$I=\frac{x\cos^{-1}x}{2}-\frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2}-\frac{1}{2}\cos^{-1}x\right)-\frac{1}{2}\cos^{-1}x$$

$$=\frac{\left(2x^2-1\right)}{4}\cos^{-1}x-\frac{x}{4}\sqrt{1-x^2}+C$$

Question 10: $(\sin^{-1}x)^2$

Answer 10:

Let $I = \int (\sin^{-1}x)^2 \cdot 1\, dx$

Taking $(\sin^{-1}x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I = (\sin^{-1}x)\int 1\,dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)^2 \cdot \int 1\cdot dx\right\}dx$$

$$= (\sin^{-1}x)^2 \cdot x - \int \frac{2\sin^{-1}x}{\sqrt{1-x^2}} \cdot x\,dx$$

$$= x(\sin^{-1}x)^2 + \int \sin^{-1}x \cdot \left(\frac{-2x}{\sqrt{1-x^2}}\right)dx$$

$$= x(\sin^{-1}x)^2 + \left[\sin^{-1}x \int \frac{-2x}{\sqrt{1-x^2}}dx - \int\left\{\left(\frac{d}{dx}\sin^{-1}x\right)\int \frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$

$$= x(\sin^{-1}x)^2 + \left[\sin^{-1}x \cdot 2\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2}\,dx\right]$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\sin^{-1}x - \int 2\,dx$$

$$= x(\sin^{-1}x)^2 + 2\sqrt{1-x^2}\sin^{-1}x - 2x + C$$

Question 11: $\frac{x\cos^{-1}x}{\sqrt{1-x^2}}$

Answer 11:

Let $I = \int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx$

$$I = \frac{-1}{2}\int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1}x\,dx$$

Taking $\cos^{-1}x$ as first function and $\left(\frac{-2x}{\sqrt{1-x^2}}\right)$ as second function and integrating by parts, we obtain

$$I=\frac{-1}{2}\left[\cos^{-1}x\int\frac{-2x}{\sqrt{1-x^2}}dx-\int\left\{\left(\frac{d}{dx}\cos^{-1}x\right)\int\frac{-2x}{\sqrt{1-x^2}}dx\right\}dx\right]$$
$$=\frac{-1}{2}\left[\cos^{-1}x\cdot2\sqrt{1-x^2}-\int\frac{-1}{\sqrt{1-x^2}}\cdot2\sqrt{1-x^2}\,dx\right]$$
$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+\int2\,dx\right]$$
$$=\frac{-1}{2}\left[2\sqrt{1-x^2}\cos^{-1}x+2x\right]+C$$
$$=-\left[\sqrt{1-x^2}\cos^{-1}x+x\right]+C$$

Question 12: $x\sec^2x$

Answer 12:

Let $I=\int x\sec^2x\,dx$

Taking x as first function and $\sec^2x$ as second function and integrating by parts, we obtain

$$I=x\int\sec^2x\,dx-\int\left\{\left\{\frac{d}{dx}x\right\}\int\sec^2x\,dx\right\}dx$$
$$=x\tan x-\int1\cdot\tan x\,dx$$
$$=x\tan x+\log|\cos x|+C$$

Question 13: $\tan^{-1}x$

Answer 13:

Let $I=\int1\cdot\tan^{-1}x\,dx$

Taking $\tan^{-1}x$ as first function and 1 as second function and integrating by parts, we obtain

$$I=\tan^{-1}x\int 1dx-\int\left\{\left(\frac{d}{dx}\tan^{-1}x\right)\int 1\cdot dx\right\}dx$$

$$=\tan^{-1}x\cdot x-\int\frac{1}{1+x^2}\cdot x\,dx$$

$$=x\tan^{-1}x-\frac{1}{2}\int\frac{2x}{1+x^2}dx$$

$$=x\tan^{-1}x-\frac{1}{2}\log\left|1+x^2\right|+C$$

$$=x\tan^{-1}x-\frac{1}{2}\log\left(1+x^2\right)+C$$

Question 14: $x(\log x)^2$

Answer 14:

$$I=\int x(\log x)^2\,dx$$

Taking $(\log x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$I=(\log x)^2\int x\,dx-\int\left[\left\{\left(\frac{d}{dx}\log x\right)^2\right\}\int x\,dx\right]dx$$

$$=\frac{x^2}{2}(\log x)^2-\left[\int 2\log x\cdot\frac{1}{x}\cdot\frac{x^2}{2}dx\right]$$

$$=\frac{x^2}{2}(\log x)^2-\int x\log x\,dx$$

Again integrating by parts, we obtain

$$I=\frac{x^2}{2}(\log x)^2-\left[\log x\int x\,dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int x\,dx\right\}dx\right]$$
$$=\frac{x^2}{2}(\log x)^2-\left[\frac{x^2}{2}-\log x-\int\frac{1}{x}\cdot\frac{x^2}{2}dx\right]$$
$$=\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{1}{2}\int x\,dx$$
$$=\frac{x^2}{2}(\log x)^2-\frac{x^2}{2}\log x+\frac{x^2}{4}+C$$

Question 15: $(x^2+1)\log x$

Answer 15:

Let $I=\int(x^2+1)\log x\,dx=\int x^2\log x\,dx+\int\log x\,dx$

Let I = I_1 + I_2 ... (1)

Where, $I_1=\int x^2\log x\,dx$ and $I_2=\int\log x\,dx$

$$I_1=\int x^2\log x\,dx$$

Taking log x as first function and x^2 as second function and integrating by parts, we obtain

$$I_1=\log x-\int x^2dx-\int\left\{\left(\frac{d}{dx}\log x\right)\int x^2dx\right\}dx$$
$$=\log x\cdot\frac{x^3}{3}-\int\frac{1}{x}\cdot\frac{x^3}{3}dx$$
$$=\frac{x^3}{3}\log x-\frac{1}{3}\left(\int x^2dx\right)$$
$$=\frac{x^3}{3}\log x-\frac{x^3}{9}+C_1 \qquad ...(2)$$
$$I_2=\int\log x\,dx$$

Taking log x as first function and 1 as second function and integrating by parts, we obtain

$$I_2 = \log x \int 1 \cdot dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 \cdot dx \right\}$$
$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$
$$= x \log x - \int 1 dx$$
$$= x \log x - x + C_2 \qquad \dots (3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$
$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + x \log x - x + (C_1 + C_2)$$
$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

Question 16: $e^x(\sin x + \cos x)$

Answer 16:

Let $I = \int e^x (\sin x + \cos x) dx$

Let $f(x) = \sin x$

$f'(x) = \cos x$

$$I = \int e^x \{ f(x) + f'(x) \} dx$$

It is known that, $\int e^x \{ f(x) + f'(x) \} dx = e^x f(x) + C$

$\therefore I = e^x \sin x + C$

Question 17: $\dfrac{xe^x}{(1+x)^2}$

Answer 17:

Let $I = \int \dfrac{xe^x}{(1+x)^2}dx = \int e^x \left\{\dfrac{x}{(1+x)^2}\right\} dx$

$= \int e^x \left\{\dfrac{1+x-1}{(1+x)^2}\right\} dx$

$= \int e^x \left\{\dfrac{1}{1+x} - \dfrac{1}{(1+x)^2}\right\} dx$

Let $f(x) = \dfrac{1}{1+x}$ $f'(x) = \dfrac{-1}{(1+x)^2}$

$\Rightarrow \int \dfrac{xe^x}{(1+x)^2}dx = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$\therefore \int \dfrac{xe^x}{(1+x)^2} dx = \dfrac{e^x}{1+x} + C$

Question 18: $e^x\left(\dfrac{1+\sin x}{1+\cos x}\right)$

Answer 18: $e^x\left(\dfrac{1+\sin x}{1+\cos x}\right)$

$$=e^x\left(\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$=\frac{e^x\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2}e^x\cdot\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}}\right)^2$$

$$=\frac{1}{2}e^x\left[\tan\frac{x}{2}+1\right]^2$$

$$=\frac{1}{2}e^2\left(1+\tan\frac{x}{2}\right)^2$$

$$=\frac{1}{2}e^x\left[1+\tan^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$=\frac{1}{2}e^x\left[\sec^2\frac{x}{2}+2\tan\frac{x}{2}\right]$$

$$\frac{e^x(1+\sin x)dx}{(1+\cos x)}=e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right] \qquad \ldots(1)$$

Let $\tan\frac{x}{2}=f(x)$ So $f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

From equation (1), we obtain

$$\int\frac{e^x(1+\sin x)}{(1+\cos x)}dx=e^x\tan\frac{x}{2}+C$$

Question 19: $e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Answer 19:

Let $I=\int e^x\left[\frac{1}{x}-\frac{1}{x^2}\right]dx$

Also, let $\frac{1}{x}=f(x)$ $\quad f'(x)=\frac{-1}{x^2}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

$\therefore I=\frac{e^x}{x}+C$

Question 20: $\frac{(x-3)e^x}{(x-1)^3}$

Answer 20:

$$\int e^x\left\{\frac{x-3}{(x-1)^3}\right\}dx=\int e^x\left\{\frac{x-1-2}{(x-1)^3}\right\}dx$$

$$=\int e^x\left\{\frac{1}{(x-1)^2}-\frac{2}{(x-1)^3}\right\}dx$$

Let $f(x)=\frac{1}{(x-1)^2}$ $\quad f'(x)=\frac{-2}{(x-1)^3}$

It is known that, $\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+C$

$$\therefore \int e^x\left\{\frac{(x-3)}{(x-1)^2}\right\}dx=\frac{e^x}{(x-1)^2}+C$$

Question 21: $e^{2x}\sin x$

Answer 21:

Let $I=\int e^{2x}\sin x\,dx$ ...(1)

Integrating by parts, we obtain

$$I=\sin x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\sin x\right)\int e^{2x}dx\right\}dx$$

$$\Rightarrow I=\sin x\cdot\frac{e^{2x}}{2}-\int\cos x\cdot\frac{e^{2x}}{2}dx$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\int e^{2x}\cos x\,dx$$

Again integrating by parts, we obtain

$$I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\cos x\int e^{2x}dx-\int\left\{\left(\frac{d}{dx}\cos x\right)\int e^{2x}dx\right\}dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{1}{2}\left[\cos x\cdot\frac{e^{2x}}{2}-\int(-\sin x)\frac{e^{2x}}{2}dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\cdot\sin x}{2}-\frac{1}{2}\left[\frac{e^{2x}\cos x}{2}+\frac{1}{2}\int e^{2x}\sin x dx\right]$$

$$\Rightarrow I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}-\frac{1}{4}I \quad \text{[From (1)]}$$

$$\Rightarrow I+\frac{1}{4}I=\frac{e^{2x}\cdot\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow \frac{5}{4}I=\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}$$

$$\Rightarrow I=\frac{4}{5}\left[\frac{e^{2x}\sin x}{2}-\frac{e^{2x}\cos x}{4}\right]+C$$

$$\Rightarrow I=\frac{e^{2x}}{5}\left[2\sin x-\cos x\right]+C$$

Question 22: $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Answer 22:

Let $x = \tan\theta$ $\quad dx = \sec^2\theta\, d\theta$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \int 2\theta \cdot \sec^2\theta\, d\theta = 2\int \theta \cdot \sec^2\theta\, d\theta$$

Integrating by parts, we obtain

$$2\left[\theta \cdot \int \sec^2\theta d\theta - \int\left\{\left(\frac{d}{d\theta}\theta\right)\int \sec^2\theta d\theta\right\}d\theta\right]$$

$$= 2\left[\theta \cdot \tan\theta - \int \tan\theta d\theta\right]$$

$$= 2\left[\theta\tan\theta + \log|\cos\theta|\right] + C$$

$$= 2\left[x\tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C$$

$$= 2x\tan^{-1}x + 2\log(1+x^2)^{-\frac{1}{2}} + C$$

$$= 2x\tan^{-1}x + 2\left[-\frac{1}{2}\log(1+x^2)\right] + C$$

$$= 2x\tan^{-1}x - \log(1+x^2) + C$$

Question 23:

$\int x^2 e^{x^3} dx$ equals

(A) $\frac{1}{3}e^{x^3} + C$ (B) $\frac{1}{3}e^{x^2} + C$

(C) $\frac{1}{2}e^{x^3} + C$ (D) $\frac{1}{3}e^{x^2} + C$

Answer 23:

Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t$ so $3x^2 dx = dt$

$\Rightarrow I = \frac{1}{3}\int e^t dt$

$= \frac{1}{3}(e^t) + C$

$= \frac{1}{3}e^{x^3} + C$

Hence, the correct Answer is A.

Question 24:

$\int e^x \sec x(1+\tan x)dx$ equals

(A) $e^x \cos x + C$ (B) $e^x \sec x + C$

(C) $e^x \sin x + C$ (D) $e^x \tan x + C$

Answer 24:

$\int e^x \sec x(1+\tan x)dx$

Let $I = \int e^x \sec x(1+\tan x)dx = \int e^x(\sec x + \sec x \tan x)dx$

Also, let $\sec x = f(x)$ $\sec x \tan x = f'(x)$

It is known that, $\int e^x\{f(x)+f'(x)\}dx = e^x f(x) + C$

$\therefore I = e^x \sec x + C$

Hence, the correct Answer is B.