Chapter - 9 Differential Equations

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Exercise 9.2

Question 1:

$$y = e^x + 1$$
 : $y'' - y' = 0$

Answer

 $y = e^{x} + 1$

Differentiating both sides of this equation with respect to x, we get:

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$$\frac{dy}{dx} = \frac{d}{dx} \left(e^x + 1 \right)$$
$$\Rightarrow v' = e^x$$

Now, differentiating equation (1) with respect to x, we get:

$$\frac{d}{dx}(y') = \frac{d}{dx}(e^x)$$
$$\Rightarrow y'' = e^x$$

Substituting the values of y' and y'' in the given differential equation, we get the L.H.S. as:

$$v'' - v' = e^x - e^x = 0 = \text{R.H.S.}$$

Thus, the given function is the solution of the corresponding differential equation.

Question 2:

$$y = x^2 + 2x + C \qquad : y'$$

Answer

 $y = x^2 + 2x + C$

Differentiating both sides of this equation with respect to x, we get:

-2x-2=0

$$y' = \frac{d}{dx} \left(x^2 + 2x + C \right)$$
$$\Rightarrow y' = 2x + 2$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = y' - 2x - 2 = 2x + 2 - 2x - 2 = 0

= R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

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Question 3: $y = \cos x + C$: $y' + \sin x = 0$

Answer

$$y = \cos x + C$$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx} (\cos x + C)$$
$$\Rightarrow y' = -\sin x$$

Substituting the value of y' in the given differential equation, we get:

L.H.S. = $y' + \sin x = -\sin x + \sin x = 0$

 $v = \sqrt{1+x}$

Hence, the given function is the solution of the corresponding differential equation.

= R.H.S.

Question 4:

Answer

 $y = \sqrt{1 + x^2}$

Differentiating both sides of the equation with respect to x, we get:

$$y' = \frac{d}{dx} \left(\sqrt{1 + x^2} \right)$$

$$y' = \frac{1}{2\sqrt{1 + x^2}} \cdot \frac{d}{dx} \left(1 + x^2 \right)$$

$$y' = \frac{2x}{2\sqrt{1 + x^2}}$$

$$y' = \frac{x}{\sqrt{1 + x^2}}$$

$$\Rightarrow y' = \frac{x}{1 + x^2} \times \sqrt{1 + x^2}$$

$$\Rightarrow y' = \frac{x}{1 + x^2} \cdot y$$

$$\Rightarrow y' = \frac{xy}{1 + x^2}$$

Hence, the given function is the solution of the corresponding differential equation.

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Question 5:

$$y = Ax$$
 : $xy' = y(x \neq 0)$

Answer

y = Ax

Differentiating both sides with respect to x, we get:

$$y' = \frac{d}{dx} (Ax)$$
$$\Rightarrow y' = A$$

Substituting the value of y' in the given differential equation, we get: L.H.S. = $xy' = x \cdot A = Ax = y = R.H.S.$

Hence, the given function is the solution of the corresponding differential equation.

Question 6:

$$y = x \sin x$$
 : $xy' = y + x\sqrt{x^2 - y^2}$ ($x \neq 0$ and $x > y$ or $x < -y$)

Answer

 $y = x \sin x$

Differentiating both sides of this equation with respect to x, we get:

$$y' = \frac{d}{dx}(x\sin x)$$

$$\Rightarrow y' = \sin x \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\sin x)$$

$$\Rightarrow y' = \sin x + x\cos x$$

Substituting the value of y'

in the given differential equation, we get:

L.H.S. =
$$xy' = x(\sin x + x \cos x)$$

= $x \sin x + x^2 \cos x$
= $y + x^2 \cdot \sqrt{1 - \sin^2 x}$
= $y + x^2 \sqrt{1 - \left(\frac{y}{x}\right)^2}$
= $y + x \sqrt{y^2 - x^2}$
= R.H.S.



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Hence, the given function is the solution of the corresponding differential equation.

Question 7:

$$xy = \log y + C$$
 : $y' = \frac{y^2}{1 - xy} (xy \neq 1)$

Answer

 $xy = \log y + C$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{d}{dx}(xy) = \frac{d}{dx}(\log y)$$

$$\Rightarrow y \cdot \frac{d}{dx}(x) + x \cdot \frac{dy}{dx} = \frac{1}{y} \frac{dy}{dx}$$

$$\Rightarrow y + xy' = \frac{1}{y} y'$$

$$\Rightarrow y^{2} + xy y' = y'$$

$$\Rightarrow (xy - 1) y' = -y^{2}$$

$$\Rightarrow y' = \frac{y^{2}}{1 - xy}$$

$$\therefore L.H.S. = R.H.S.$$

Hence, the given function is the solution of the corresponding differential equation.

Question 8:

$$y - \cos y = x \qquad \qquad : \quad (y \sin y + \cos y + x)y' = y$$

Answer

 $y - \cos y = x$

...(1)

Differentiating both sides of the equation with respect to x, we get:

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$$\frac{dy}{dx} - \frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$$
$$\Rightarrow y' + \sin y \cdot y' = 1$$
$$\Rightarrow y'(1 + \sin y) = 1$$
$$\Rightarrow y' = \frac{1}{1 + \sin y}$$

Substituting the value of y' in

in equation (1), we get:

L.H.S. =
$$(y \sin y + \cos y + x)y'$$

= $(y \sin y + \cos y + y - \cos y) \times \frac{1}{1 + \sin y}$
= $y(1 + \sin y) \cdot \frac{1}{1 + \sin y}$
= y
= R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 9:

$$x + y = \tan^{-1} y$$

Answer

 $x + y = \tan^{-1} y$

Differentiating both sides of this equation with respect to x, we get:

: $y^2y' + y^2 + 1 = 0$

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$$\frac{d}{dx}(x+y) = \frac{d}{dx}(\tan^{-1}y)$$
$$\Rightarrow 1+y' = \left[\frac{1}{1+y^2}\right]y'$$
$$\Rightarrow y'\left[\frac{1}{1+y^2}-1\right] = 1$$
$$\Rightarrow y'\left[\frac{1-(1+y^2)}{1+y^2}\right] = 1$$
$$\Rightarrow y'\left[\frac{-y^2}{1+y^2}\right] = 1$$
$$\Rightarrow y' = \frac{-(1+y^2)}{y^2}$$

V/On

Substituting the value of \mathcal{V} in the given differential equation, we get:

L.H.S. =
$$y^2y' + y^2 + 1 = y^2 \left[\frac{-(1+y^2)}{y^2} \right] + y^2 + 1$$

= $-1 - y^2 + y^2 + 1$
= 0
= R.H.S.

Hence, the given function is the solution of the corresponding differential equation.

Question 10:

$$y = \sqrt{a^2 - x^2} x \in (-a, a)$$
 : $x + y \frac{dy}{dx} = 0 (y \neq 0)$

Answer

$$y = \sqrt{a^2 - x^2}$$

Differentiating both sides of this equation with respect to x, we get:

$$\frac{dy}{dx} = \frac{d}{dx} \left(\sqrt{a^2 - x^2} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}} \cdot \frac{d}{dx} \left(a^2 - x^2 \right)$$

$$= \frac{1}{2\sqrt{a^2 - x^2}} (-2x)$$

$$= \frac{-x}{\sqrt{a^2 - x^2}}$$
Substituting the value of
L.H.S. = $x + y \frac{dy}{dx} = x + \sqrt{a^2 - x^2} \times \frac{-x}{\sqrt{a^2 - x^2}}$

$$= x - x$$

$$= 0$$

$$= R.H.S.$$

Hence, the given function is the solution of the corresponding differential equation.

Question 11:

The numbers of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4

Answer

We know that the number of constants in the general solution of a differential equation of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Hence, the correct answer is D.

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Question 12:

The numbers of arbitrary constants in the particular solution of a differential equation of third order are:

(Δ) 3	(B) 2	(C) 1	(ח) ח
(A) J			(D) 0

Answer

In a particular solution of a differential equation, there are no arbitrary constants. Hence, the correct answer is D.

