Exercise 10.2

Question 1:

Compute the magnitude of the following vectors:

 $\vec{a} = \hat{i} + \hat{j} + \hat{k}; \quad \vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}; \qquad \vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$

Answer 1:

The given vectors are:



Question 2:

Write two different vectors having same magnitude.

Answer 2:

Consider $\vec{a} = (\hat{i} - 2\hat{j} + 3\hat{k})$ and $\vec{b} = (2\hat{i} + \hat{j} - 3\hat{k})$. It can be observed that $|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$ and $|\vec{b}| = \sqrt{2^2 + 1^2 + (-3)^2} = \sqrt{4 + 1 + 9} = \sqrt{14}$.

Hence, \vec{a} and \vec{b} are two different vectors having the same magnitude. The vectors are different because they have different directions.

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Question 3:

Write two different vectors having same direction.

Answer 3:

Consider $\vec{p} = (\hat{i} + \hat{j} + \hat{k})$ and $\vec{q} = (2\hat{i} + 2\hat{j} + 2\hat{k})$.

The direction cosines of \vec{p} are given by,

$$l = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \ m = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \ \text{and} \ n = \frac{1}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}.$$

The direction cosines of \vec{q} are given by

$$l = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, m = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$$

and $n = \frac{2}{\sqrt{2^2 + 2^2 + 2^2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}},$

The direction cosines of \vec{p} and \vec{q} are the same. Hence, the two vectors have the same direction.

Question 4:

Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal

Answer 4:

The two vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ will be equal if their corresponding components are equal.

Hence, the required values of x and y are 2 and 3 respectively.

Question 5:

Find the scalar and vector components of the vector with initial point (2, 1) and terminal point (-5, 7).

Answer 5:

The vector with the initial point P (2, 1) and terminal point Q (-5, 7) can be given by, $\overrightarrow{PQ} = (-5-2)\hat{i} + (7-1)\hat{j}$ $\Rightarrow \overrightarrow{PQ} = -7\hat{i} + 6\hat{j}$

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Hence, the required scalar components are -7 and 6 while the vector components are $-7\hat{i}$ and $6\hat{j}$.

Question 6:

Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Answer 6:

The given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

$$\therefore \vec{a} + \vec{b} + \vec{c} = (1 - 2 + 1)\hat{i} + (-2 + 4 - 6)\hat{j} + (1 + 5 - 7)\hat{k}$$

= $0 \cdot \hat{i} - 4\hat{j} - 1 \cdot \hat{k}$
= $-4\hat{j} - \hat{k}$
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Question 7:

Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2k$

Answer 7:

The unit vector \hat{a} in the direction of vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ $|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$ $\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$

Question 8:

Find the unit vector in the direction of vector \overline{PQ} , where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Answer 8:

The given points are P (1, 2, 3) and Q (4, 5, 6).

$$\therefore \overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$
$$\left|\overrightarrow{PQ}\right| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{9+9+9} = \sqrt{27} = 3\sqrt{3}$$

Hence, the unit vector in the direction of \overrightarrow{PQ} is

$$\frac{\overline{PQ}}{|\overline{PQ}|} = \frac{3\hat{i}+3\hat{j}+3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Question 9:

For given vectors, $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction

of the vector $\vec{a} + \vec{b}$

Answer 9:

The given vectors are $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$. $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$ $\therefore \vec{a} + \vec{b} = (2-1)\hat{i} + (-1+1)\hat{j} + (2-1)\hat{k} = 1\hat{i} + 0\hat{j} + 1\hat{k} = \hat{i} + \hat{k}$ $\left|\vec{a} + \vec{b}\right| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence, the unit vector in the direction of $\left(\vec{a}+\vec{b}\right)$ is

$$\frac{\left(\vec{a}+\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|} = \frac{\hat{i}+\hat{k}}{\sqrt{2}} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$$

Question 10:

Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units. **Answer 10**:

Let
$$\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$$
.
 $\therefore |\vec{a}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25 + 1 + 4} = \sqrt{30}$
 $\therefore \hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}$

Hence, the vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units is given by,

$$8\hat{a} = 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right) = \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$

$$= 8\left(\frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}\right)$$

$$= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$$
Question 11:
Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.
Answer 11:
Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$.
It is observed that $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k} = -2(2\hat{i} - 3\hat{j} + 4\hat{k}) = -2\vec{a}$
 $\therefore \vec{b} = \lambda \vec{a}$
where,
 $\lambda = -2$

Hence, the given vectors are collinear.

Question 12:

Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$ Answer 12:

Let
$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$
.
 $\therefore |\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

Hence, the direction cosines of

$$\vec{a}$$
 are $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

Question 13:

Find the direction cosines of the vector joining the points A (1, 2, -3) and

B (-1, -2, 1) directed from A to B.

Answer 13:

The given points are A (1, 2, -3) and B (-1, -2, 1).

$$\therefore \overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + \{1-(-3)\}\hat{k}$$

$$\Rightarrow \overrightarrow{AB} = -2\hat{i} - 4\hat{j} + 4\hat{k}$$

$$\therefore |\overrightarrow{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

Hence, the direction cosines of \overline{AB} are

Question 14:

Show that the vector $\hat{i} + \hat{j} + \hat{k}$ is equally inclined to the axes OX, OY, and OZ.

Answer 14:

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. Then, $|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

Therefore, the direction cosines of

$$\vec{a}$$
 are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

 $\frac{2}{3}, \frac{2}{3}$

Now, let α , β , and γ be the angles formed by \vec{a} with the positive directions of x, y, and z axes.

Then, we have

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$$\cos\alpha = \frac{1}{\sqrt{3}}, \cos\beta = \frac{1}{\sqrt{3}}, \cos\gamma = \frac{1}{\sqrt{3}}.$$

Hence, the given vector is equally inclined to axes OX, OY, and OZ.

Question 15:

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ration 2:1

(i) internally

(ii)externally

Answer 15:

ational S

The position vector of point R dividing the line segment joining two points P and Q in the ratio m: n is given by:

i. Internally: $\frac{m\vec{b} + n\vec{a}}{m+n}$ ii. Externally: $\frac{m\vec{b} - n\vec{a}}{m-n}$ Position vectors of P and Q are given as:

 $\overrightarrow{OP} = \hat{i} + 2\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + \hat{j} + \hat{k}$

(i) The position vector of point R which divides the line joining two points P and Q internally in the ratio 2:1 is given by,

$$\overline{OR} = \frac{2(-\hat{i}+\hat{j}+\hat{k})+1(\hat{i}+2\hat{j}-\hat{k})}{2+1} = \frac{(-2\hat{i}+2\hat{j}+2\hat{k})+(\hat{i}+2\hat{j}-\hat{k})}{3} = \frac{-\hat{i}+4\hat{j}+\hat{k}}{3} = -\frac{1}{3}\hat{i}+\frac{4}{3}\hat{j}+\frac{1}{3}\hat{k}$$

(ii) The position vector of point R which divides the line joining two points P and Q externally in the ratio 2:1 is given by,

$$\overline{\text{OR}} = \frac{2\left(-\hat{i} + \hat{j} + \hat{k}\right) - 1\left(\hat{i} + 2\hat{j} - \hat{k}\right)}{2 - 1} = \left(-2\hat{i} + 2\hat{j} + 2\hat{k}\right) - \left(\hat{i} + 2\hat{j} - \hat{k}\right)$$
$$= -3\hat{i} + 3\hat{k}$$

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Question 16:

Find the position vector of the mid point of the vector joining the points P (2, 3, 4) and

Q (4, 1, - 2).

Answer 16:

The position vector of mid-point R of the vector joining points P (2, 3, 4) and Q (4, 1, –

$$\overline{OR} = \frac{\left(2\hat{i}+3\hat{j}+4\hat{k}\right) + \left(4\hat{i}+\hat{j}-2\hat{k}\right)}{2} = \frac{(2+4)\hat{i}+(3+1)\hat{j}+(4-2)\hat{k}}{2}$$
$$= \frac{6\hat{i}+4\hat{j}+2\hat{k}}{2} = 3\hat{i}+2\hat{j}+\hat{k}$$

Question 17:

Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$,

respectively form the vertices of a right angled triangle. Answer 17:

Position vectors of points A, B, and C are respectively given as: $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \ \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \ \text{and} \ \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}, \ \vec{b} = 2\hat{i} - \hat{j} + \hat{k} \ \text{and} \ \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$ $\therefore \overrightarrow{AB} = \vec{b} - \vec{a} = (2 - 3)\hat{i} + (-1 + 4)\hat{j} + (1 + 4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}$ $\overrightarrow{BC} = \vec{c} - \vec{b} = (1 - 2)\hat{i} + (-3 + 1)\hat{j} + (-5 - 1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}$ $\overrightarrow{CA} = \vec{a} - \vec{c} = (3 - 1)\hat{i} + (-4 + 3)\hat{j} + (-4 + 5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}$ $\therefore |\overrightarrow{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35$ $|\overrightarrow{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41$ $|\overrightarrow{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6$ $\therefore |\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = 36 + 6 = 41 = |\overrightarrow{BC}|^2$

Hence, ABC is a right-angled triangle.

Question 18:

In triangle ABC which of the following is not true:



On applying the triangle law of addition in the given triangle, we have:

 $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$...(1) $\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$ $\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{0}$...(2) . The equation given in alternative A is true. $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ $\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \overrightarrow{0}$... The equation given in alternative B is true. From equation (2), we have: $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \overrightarrow{0}$... The equation given in alternative D is true. Now, consider the equation given in alternative C: $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \overrightarrow{0}$ $\Rightarrow AB + BC = CA$...(3) From equations (1) and (3), we have: $\overrightarrow{AC} = \overrightarrow{CA}$ $\Rightarrow \overrightarrow{AC} = -\overrightarrow{AC}$ $\Rightarrow \overrightarrow{AC} + \overrightarrow{AC} = \overrightarrow{0}$ $\Rightarrow 2\overrightarrow{AC} = \overrightarrow{0}$

 $\Rightarrow \overrightarrow{AC} = \overrightarrow{0}$, which is not true.

Hence, the equation given in alternative C is incorrect.

The correct answer is C.

Question 19:

If \vec{a} and \vec{b} are two collinear vectors, then which of the following are incorrect:

A. $\vec{b} = \lambda \vec{a}$, for some scalar λ

B.
$$\vec{a} = \pm \vec{b}$$

C. the respective components of \vec{a} and \vec{b} are proportional

D. both the vectors \vec{a} and \vec{b} have same direction, but different magnitudes

Answer 19:

The correct answer is D.

If \vec{a} and \vec{b} are two collinear vectors, then they are parallel. Therefore, we have: $\vec{b} = \lambda \vec{a}$ (For some scalar λ) If $\lambda = \pm 1$, then $\vec{a} = \pm \vec{b}$. If $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$ and $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$, then $\vec{b} = \lambda \vec{a}$. $\Rightarrow b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k} = \lambda (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k})$ $\Rightarrow b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k} = (\lambda a_1) \hat{i} + (\lambda a_2) \hat{j} + (\lambda a_3) \hat{k}$ on al $\Rightarrow b_1 = \lambda a_1, b_2 = \lambda a_2, b_3 = \lambda a_3$ $\Rightarrow \frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \lambda$ Thus, the respective components of \vec{a} and \vec{b} are proportional. However, vectors \vec{a} and \vec{b} can have different directions. Hence, the statement given in D is incorrect.