Exercise 13.1

Question 1:

Given that E and F are events such that P(E) = 0.6, P(F) = 0.3 and P(E ∩ F) = 0.2, find P (E|F) and P(F|E).

Answer 1:

It is given that P(E) = 0.6, P(F) = 0.3, and P(E ∩ F) = 0.2

$$\Rightarrow P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$\Rightarrow P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

Question 2:

Compute P(A|B), if P(B) = 0.5 and P (A ∩ B) = 0.32

Answer 2:

It is given that P(B) = 0.5 and P(A ∩ B) = 0.32

$$\Rightarrow P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.32}{0.5} = \frac{16}{25}$$

Question 3:

If P(A) = 0.8, P(B) = 0.5 and P(B|A) = 0.4, find

(i) P(A ∩ B) (ii) P(A|B) (iii) P(A ∪ B)

Answer 3:

It is given that P(A) = 0.8, P(B) = 0.5, and P(B|A) = 0.4

(i) P (B|A) = 0.4

$$\therefore \frac{P(A \cap B)}{P(A)} = 0.4$$

$$\Rightarrow \frac{P(A \cap B)}{0.8} = 0.4$$

$$\Rightarrow P(A \cap B) = 0.32$$

(ii) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow P(A|B) = \frac{0.32}{0.5} = 0.64$$

(iii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.5 + 0.5 - 0.32 = 0.98$$

Question 4:

Evaluate P (A ∪ B), if 2P (A) = P (B) = $\frac{5}{13}$ and P(A|B) = $\frac{2}{5}$

Answer 4:

It is given that, $2P(A)=P(B)=\frac{5}{13}$

$\Rightarrow P(A)=\frac{5}{26}$ and $P(B)=\frac{5}{13}$

$P(A|B)=\frac{2}{5}$

$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{2}{5}$

$\Rightarrow P(A\cap B)=\frac{2}{5}\times P(B)=\frac{2}{5}\times\frac{5}{13}=\frac{2}{13}$

It is known that, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow P(A\cup B)=\frac{5}{26}+\frac{5}{13}-\frac{2}{13}$

$\Rightarrow P(A\cup B)=\frac{5+10-4}{26}$

$\Rightarrow P(A\cup B)=\frac{11}{26}$

Question 5:

If P(A) = $\frac{6}{11}$, P(B) = $\frac{5}{11}$ and P(A ∪ B) = $\frac{7}{11}$, find

(i) P(A ∩ B) (ii) P(A|B) (iii) P(B|A)

Answer 5:

It is given that $P(A)=\frac{6}{11}, P(B)=\frac{5}{11}$, and $P(A\cup B)=\frac{7}{11}$

(i) $P(A\cup B)=\frac{7}{11}$

$\therefore P(A)+P(B)-P(A\cap B)=\frac{7}{11}$

$\Rightarrow \frac{6}{11}+\frac{5}{11}-P(A\cap B)=\frac{7}{11}$

$\Rightarrow P(A\cap B)=\frac{11}{11}-\frac{7}{11}=\frac{4}{11}$

(ii) It is known that, $P(A|B)=\frac{P(A\cap B)}{P(B)}$

$$\Rightarrow P(A|B)=\frac{\frac{4}{11}}{\frac{5}{11}}=\frac{4}{5}$$

(iii) It is known that, $P(B|A)=\frac{P(A\cap B)}{P(A)}$

$$\Rightarrow P(B|A)=\frac{\frac{4}{11}}{\frac{6}{11}}=\frac{4}{6}=\frac{2}{3}$$

Question 6:

A coin is tossed three times, where

(i) E: head on third toss, F: heads on first two tosses

(ii) E: at least two heads, F: at most two heads

(iii) E: at most two tails, F: at least one tail

Answer 6:

If a coin is tossed three times, then the sample space S is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that the sample space has 8 elements.

(i) E = {HHH, HTH, THH, TTH}

F = {HHH, HHT}

$\therefore E\cap F$ = {HHH}

$P(F)=\frac{2}{8}=\frac{1}{4}$ and $P(E\cap F)=\frac{1}{8}$

$$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{1}{8}}{\frac{1}{4}}=\frac{4}{8}=\frac{1}{2}$$

(ii) E = {HHH, HHT, HTH, THH}

F = {HHT, HTH, HTT, THH, THT, TTH, TTT}

$\therefore E\cap F$ = {HHT, HTH, THH}

Clearly, $P(E\cap F)=\frac{3}{8}$ and $P(F)=\frac{7}{8}$

$$P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{\frac{3}{8}}{\frac{7}{8}}=\frac{3}{7}$$

(iii) $E = \{HHH, HHT, HTT, HTH, THH, THT, TTH\}$

$F = \{HHT, HTT, HTH, THH, THT, TTH, TTT\}$

$\therefore E \cap F = \{HHT, HTT, HTH, THH, THT, TTH\}$

$P(F) = \frac{7}{8}$ and $P(E \cap F) = \frac{6}{8}$

Therefore, $P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{6}{8}}{\frac{7}{8}} = \frac{6}{7}$

Question 7:

Two coins are tossed once, where

(i) E: tail appears on one coin, F: one coin shows head

(ii) E: not tail appears, F: no head appears

Answer 7:

If two coins are tossed once, then the sample space S is

$S = \{HH, HT, TH, TT\}$

(i) $E = \{HT, TH\}$

$F = \{HT, TH\}$

$\therefore E \cap F = \{HT, TH\}$

$P(F) = \frac{2}{8} = \frac{1}{4}$

$P(E \cap F) = \frac{2}{8} = \frac{1}{4}$

$\therefore\ P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{2}{2} = 1$

(ii) $E = \{HH\}$

$F = \{TT\}$

$\therefore E \cap F = \Phi$

$P(F) = 1$ and $P(E \cap F) = 0$

$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{0}{1} = 0$

Question 8:

A die is thrown three times,

E: 4 appears on the third toss, F: 6 and 5 appears respectively on first two tosses

Answer 8:

If a die is thrown three times, then the number of elements in the sample space will be $6 \times 6 \times 6 = 216$

$$E = \begin{Bmatrix} (1,1,4),(1,2,4),...(1,6,4) \\ (2,1,4),(2,2,4),...(2,6,4) \\ (3,1,4),(3,2,4),...(3,6,4) \\ (4,1,4),(4,2,4),...(4,6,4) \\ (5,1,4),(5,2,4),...(5,6,4) \\ (6,1,4),(6,2,4),...(6,6,4) \end{Bmatrix}$$

$$F = \{(6,5,1),(6,5,2),(6,5,3),(6,5,4),(6,5,5),(6,5,6)\}$$

$$\therefore E \cap F = \{(6,5,4)\}$$

$$P(F) = \frac{6}{216} \text{ and } P(E \cap F) = \frac{1}{216}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

Question 9:

Mother, father and son line up at random for a family picture

E: son on one end, F: father in middle

Answer 9:

If mother (M), father (F), and son (S) line up for the family picture, then the sample space will be

S = {MFS, MSF, FMS, FSM, SMF, SFM}

⇒ E = {MFS, FMS, SMF, SFM}

F = {MFS, SFM}

∴ E ∩ F = {MFS, SFM}

$$P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$$

Question 10:

A black and a red dice are rolled.

(a) Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

(b) Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

Answer 10:

Let the first observation be from the black die and second from the red die.

When two dice (one black and another red) are rolled, the sample space S has $6 \times 6 = 36$ number of elements.

1. Let

A: Obtaining a sum greater than 9

$= \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

B: Black die results in a 5.

$= \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

$\therefore A \cap B = \{(5, 5), (5, 6)\}$

The conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5, is given by P (A|B).

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{35}} = \frac{2}{6} = \frac{1}{3}$$

(b) E: Sum of the observations is 8.

$= \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

F: Red die resulted in a number less than 4.

$$= \begin{Bmatrix} (1,1),(1,2),(1,3),(2,1),(2,2),(2,3), \\ (3,1),(3,2),(3,3),(4,1),(4,2),(4,3), \\ (5,1),(5,2),(5,3),(6,1),(6,2),(6,3) \end{Bmatrix}$$

$\therefore E \cap F = \{(5,3),(6,2)\}$

$P(F) = \frac{18}{36}$ and $P(E \cap F) = \frac{2}{36}$

The conditional probability of obtaining the sum equal to 8, given that the red die resulted in a number less than 4, is given by P (E|F)

$$\text{Therefore, } P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{2}{36}}{\frac{18}{36}} = \frac{2}{18} = \frac{1}{9}$$

Question 11:

A fair die is rolled. Consider events $E = \{1, 3, 5\}$, $F = \{2, 3\}$ and $G = \{2, 3, 4, 5\}$

Find

(i) P (E|F) and P (F|E) (ii) P (E|G) and P (G|E)

(ii) P ((E ∪ F)|G) and P ((E ∩ G)|G)

Answer 11:

When a fair die is rolled, the sample space S will be

$S = \{1, 2, 3, 4, 5, 6\}$

It is given that $E = \{1, 3, 5\}$, $F = \{2, 3\}$, and $G = \{2, 3, 4, 5\}$

$$\therefore P(E) = \frac{3}{6} = \frac{1}{2}$$

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

$$P(G) = \frac{4}{6} = \frac{2}{3}$$

(i) $E \cap F = \{3\}$

$$\therefore P(E \cap F) = \frac{1}{6}$$

$$\therefore P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{6}}{\frac{1}{3}} = \frac{1}{2}$$

$$P(F|E) = \frac{P(E \cap F)}{P(E)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

(ii) $E \cap G = \{3, 5\}$

$$\therefore P(E \cap G) = \frac{2}{6} = \frac{1}{3}$$

$$\therefore P(E|G) = \frac{P(E \cap G)}{P(G)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

$$P(G|E) = \frac{P(E \cap G)}{P(E)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

(iii) $E \cup F = \{1, 2, 3, 5\}$

$(E \cup F) \cap G = \{1, 2, 3, 5\} \cap \{2, 3, 4, 5\} = \{2, 3, 5\}$

$E \cap F = \{3\}$

$(E \cap F) \cap G = \{3\} \cap \{2, 3, 4, 5\} = \{3\}$

$$\therefore P(E \cup G) = \frac{4}{6} = \frac{2}{3}$$

$$P((E \cup F) \cap G) = \frac{3}{6} = \frac{1}{2}$$

$$P(E \cap F) = \frac{1}{6}$$

$$P((E \cap F) \cap G) = \frac{1}{6}$$

$$\therefore P((E \cup F) | G) = \frac{P((E \cup F) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$P((E \cap F) | G) = \frac{P((E \cap G) \cap G)}{P(G)}$$

$$= \frac{\frac{1}{6}}{\frac{2}{3}} = \frac{1}{6} \times \frac{3}{2} = \frac{1}{4}$$

Question 12:

Assume that each born child is equally likely to be a boy or a girl. If a family has two children, what is the conditional probability that both are girls given that (i) the youngest is a girl, (ii) at least one is a girl?

Answer 12:

Let b and g represent the boy and the girl child respectively. If a family has two children, the sample space will be

$S = \{(b, b), (b, g), (g, b), (g, g)\}$

Let A be the event that both children are girls.

$\therefore A = \{(g, g)\}$

(i) Let B be the event that the youngest child is a girl.

$\therefore B = [(b,g),(g,g)]$

$\Rightarrow A \cap B = \{(g, g)\}$

$\therefore P(B) = \frac{2}{4} = \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

The conditional probability that both are girls, given that the youngest child is a girl, is given by P (A|B).

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

Therefore, the required probability is $\frac{1}{2}$.

(ii) Let C be the event that at least one child is a girl.

$\therefore C = \{(b, g),(g, b),(g, g)\}$

$\Rightarrow A \cap C = \{g, g\}$

$\Rightarrow P(C) = \frac{3}{4}$

$P(A \cap C) = \frac{1}{4}$

The conditional probability that both are girls, given that at least one child is a girl, is given by P(A|C).

$$\text{Therefore, } P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Question 13:

An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question?

Answer 13:

The given data can be tabulated as

	True/False	Multiple choice	Total
Easy	300	500	800
Difficult	200	400	600
Total	500	900	1400

Let us denote E = easy questions, M = multiple choice questions, D = difficult questions, and T = True/False questions

Total number of questions = 1400

Total number of multiple choice questions = 900

Therefore, probability of selecting an easy multiple choice question is

$$P(E \cap M) = \frac{500}{1400} = \frac{5}{14}$$

Probability of selecting a multiple choice question, P (M), is

$$\frac{900}{1400} = \frac{9}{14}$$

P (E|M) represents the probability that a randomly selected question will be an easy question, given that it is a multiple choice question.

$$\therefore P(E|M) = \frac{P(E \cap M)}{P(M)} = \frac{\frac{5}{14}}{\frac{9}{14}} = \frac{5}{9}$$

Therefore, the required probability is $\frac{5}{9}$.

Question 14:

Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Answer 14:

When dice is thrown, number of observations in the sample space = 6 × 6 = 36

Let A be the event that the sum of the numbers on the dice is 4 and B be the event that the two numbers appearing on throwing the two dice are different.

$$\therefore A = \{(1, 3), (2, 2), (3, 1)\}$$

$$B = \left\{\begin{matrix} (1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{matrix}\right\}$$

$$A \cap B = \{(1,3),(3,1)\}$$

$$\therefore P(B) = \frac{30}{36} = \frac{5}{6} \text{ and } P(A \cap B) = \frac{2}{36} = \frac{1}{18}$$

Let P (A|B) represent the probability that the sum of the numbers on the dice is 4, given that the two numbers appearing on throwing the two dice are different.

$$\therefore P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{18}}{\frac{5}{6}} = \frac{1}{15}$$

Therefore, the required probability is $\frac{1}{15}$.

Question 15:

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Answer 15:

The outcomes of the given experiment can be represented by the following tree diagram.

The sample space of the experiment is,

$$S=\left\{\begin{matrix}(1,H),(1,T),(2,H),(2,T),(3,1)(3,2),(3,3),(3,4),(3,5),(3,6),\\(4,H),(4,T),(5,H),(5,T),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{matrix}\right\}$$

Let A be the event that the coin shows a tail and B be the event that at least one die shows 3.

$$\therefore A=\{(1,T),(2,T),(4,T),(5,T)\}$$
$$B=\{(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(6,3)\}$$
$$\Rightarrow A\cap B=\phi$$
$$\therefore P(A\cap B)=0$$
$$\text{Then, } P(B)=P(\{3,1\})+P(\{3,2\})+P(\{3,3\})+P(\{3,4\})+P(\{3,5\})+P(\{3,6\})+P(\{6,3\})$$
$$=\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}+\frac{1}{36}$$
$$=\frac{7}{36}$$

Probability of the event that the coin shows a tail, given that at least one die shows 3, is given by P(A|B).

Therefore,

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{0}{\frac{7}{36}}=0$$

Question 16:

If $P(A)=\frac{1}{2}, P(B)=0,$ then $P(A|B)$ is

(A) 0 (B) $\frac{1}{2}$

(C) not defined (D) 1

Answer 16:

It is given that $P(A)=\frac{1}{2}$ and $P(B)=0$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}=\frac{P(A\cap B)}{0}$$

Therefore, P (A|B) is not defined.

Thus, the correct answer is C.

Question 17:

If A and B are events such that P (A|B) = P(B|A), then

(A) $A \subset B$ but $A \neq B$ (B) A = B

(C) $A \cap B = \Phi$ (D) P(A) = P(B)

Answer 17:

It is given that, P(A|B) = P(B|A)

$$\Rightarrow \frac{P(A\cap B)}{P(B)}=\frac{P(A\cap B)}{P(A)}$$

$\Rightarrow$ P (A) = P (B)

Thus, the correct answer is D.