Exercises

Question 5.1:

Answer the following questions regarding earth's magnetism:

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18º. Would you expect a greater or smaller dip angle in Britain?

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or South Pole?

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8 × 1022 J T^{-1} located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

Answer 5.1:

(a) The three independent quantities conventionally used for specifying earth's magnetic field are:

(i) Magnetic declination,

(ii) Angle of dip, and

(iii) Horizontal component of earth's magnetic field

(b) The angle of dip at a point depends on how far the point is located with respect to the North Pole or the South Pole. The angle of dip would be greater in Britain (it is about 70°) than in southern India because the location of Britain on the globe is closer to the magnetic North Pole.

(c) It is hypothetically considered that a huge bar magnet is dipped inside earth with its north pole near the geographic South Pole and its south pole near the geographic North Pole.

Magnetic field lines emanate from a magnetic north pole and terminate at a magnetic south pole. Hence, in a map depicting earth's magnetic field lines, the field lines at Melbourne, Australia would seem to come out of the ground.

(d) If a compass is located on the geomagnetic North Pole or South Pole, then the compass will be free to move in the horizontal plane while earth's field is exactly vertical to the magnetic poles. In such a case, the compass can point in any direction.

(e) Magnetic moment, $M = 8 \times 10^{22}$ J T^{-1}

Radius of earth, $r = 6.4 \times 10^6$ m

Magnetic field strength, $B = \frac{\mu_0 M}{4\pi r^3}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ Tm A^{-1}

$$\therefore B = \frac{4\pi \times 10^{-7} \times 8 \times 10^{22}}{4\pi \times (6.4 \times 10^6)^3} = 0.3 \text{ G}$$

This quantity is of the order of magnitude of the observed field on earth.

(f) Yes, there are several local poles on earth's surface oriented in different directions. A magnetised mineral deposit is an example of a local N-S pole.

Question 5.2:

Answer the following questions:

(a) The earth's magnetic field varies from point to point in space.

Does it also change with time? If so, on what time scale does it change appreciably?

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of 10−12 T. Can such a weak field be of any significant consequence? Explain.

[Note: Exercise 5.2 is meant mainly to arouse your curiosity. Answers to some questions above are tentative or unknown. Brief answers wherever possible are given at the end. For details, you should consult a good text on geomagnetism.]

Answer 5.2:

(a) Earth's magnetic field changes with time. It takes a few hundred years to change by an appreciable amount. The variation in earth's magnetic field with the time cannot be neglected.

(b) Earth's core contains molten iron. This form of iron is not ferromagnetic. Hence, this is not considered as a source of earth's magnetism.

(c) The radioactivity in earth's interior is the source of energy that sustains the currents in the outer conducting regions of earth's core. These charged currents are considered to be responsible for earth's magnetism.

(d) Earth reversed the direction of its field several times during its history of 4 to 5 billion years. These magnetic fields got weakly recorded in rocks during their solidification. One can get clues about the geomagnetic history from the analysis of this rock magnetism.

(e) Earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km) because of the presence of the ionosphere. In this region, earth's field gets modified because of the field of single ions. While in motion, these ions produce the magnetic field associated with them.

(f) An extremely weak magnetic field can bend charged particles moving in a circle. This may not be noticeable for a large radius path. With reference to the gigantic interstellar space, the deflection can affect the passage of charged particles.

Question 5.3:

A short bar magnet placed with its axis at 30º with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnitude of magnetic moment of the magnet?

Answer 5.3:

Magnetic field strength, B = 0.25 T

Torque on the bar magnet, T = 4.5×10^{-2} J

Angle between the bar magnet and the external magnetic field, θ = 30°

Torque is related to magnetic moment (M) as:

T = MB sin θ

$$\therefore M = \frac{T}{B\sin\theta}$$

$$= \frac{4.5 \times 10^{-2}}{0.25 \times \sin 30^\circ} = 0.36 \text{ J T}^{-1}$$

Hence, the magnetic moment of the magnet is 0.36 J T^{-1}.

Question 5.4:

A short bar magnet of magnetic moment m = 0.32 J T^{-1} is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its **(a)** stable, and **(b)** unstable equilibrium? What is the potential energy of the magnet in each case?

Answer 5.4:

Moment of the bar magnet, M = 0.32 J T^{-1}

External magnetic field, B = 0.15 T

(a) The bar magnet is aligned along the magnetic field. This system is considered as being in stable equilibrium. Hence, the angle θ, between the bar magnet and the magnetic field is 0°.

Potential energy of the system $= -MB\cos\theta$

$= -0.32 \times 0.15 \cos 0^\circ$

$= -4.8 \times 10^{-2}$ J

(b) The bar magnet is oriented 180° to the magnetic field. Hence, it is in unstable equilibrium. θ = 180°

Potential energy = − MB cos θ

$= -0.32 \times 0.15 \cos 180°$

$= 4.8 \times 10^{-2}$ J

Question 5.5:

A closely wound solenoid of 800 turns and area of cross section 2.5×10^{-4} m^2 carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Answer 5.5:

Number of turns in the solenoid, n = 800

Area of cross-section, A = 2.5×10^{-4} m^2

Current in the solenoid, I = 3.0 A

A current-carrying solenoid behaves as a bar magnet because a magnetic field develops along its axis, i.e., along its length.

The magnetic moment associated with the given current-carrying solenoid is calculated as:

M = n I A

$= 800 \times 3 \times 2.5 \times 10^{-4}$

$= 0.6$ J T^{-1}

Question 5.6:

If the solenoid in Exercise 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?

Answer 5.6:

Magnetic field strength, B = 0.25 T

Magnetic moment, M = 0.6 T^{-1}

The angle θ, between the axis of the solenoid and the direction of the applied field is 30°.

Therefore, the torque acting on the solenoid is given as:

$$\tau = MB\sin\theta$$
$$= 0.6 \times 0.25 \sin 30°$$
$$= 7.5 \times 10^{-2}\ \text{J}$$

Question 5.7:

A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment: (i) normal to the field direction, (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

Answer 5.7:

(a) Magnetic moment, M = 1.5 J T^{-1}

Magnetic field strength, B = 0.22 T

(i) Initial angle between the axis and the magnetic field, $\theta_1 = 0°$

Final angle between the axis and the magnetic field, $\theta_2 = 90°$

The work required to make the magnetic moment normal to the direction of magnetic field is given as:

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$
$$= -1.5 \times 0.22(\cos 90° - \cos 0°)$$
$$= -0.33(0-1)$$
$$= 0.33\ \text{J}$$

(ii) Initial angle between the axis and the magnetic field, $\theta_1 = 0°$

Final angle between the axis and the magnetic field, $\theta_2 = 180°$

The work required to make the magnetic moment opposite to the direction of magnetic field is given as:

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$
$$= -1.5 \times 0.22(\cos 180 - \cos 0°)$$
$$= -0.33(-1 - 1)$$
$$= 0.66\ J$$

(b) For case (i): $\theta = \theta_2 = 90°$

$\therefore$ Torque, $\tau = MB\sin\theta$

$= 1.5 \times 0.22 \sin 90°$

$= 0.33\ J$

For case (ii): $\theta = \theta_2 = 180°$

$\therefore$ Torque, $\tau = MB\sin\theta$

$= MB\sin 180° = 0\ J$

Question 5.8:

A closely wound solenoid of 2000 turns and area of cross-section $1.6 \times 10^{-4}\ m^2$, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.

(a) What is the magnetic moment associated with the solenoid?

(b) What is the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30º with the axis of the solenoid?

Answer 5.8:

Number of turns on the solenoid, n = 2000

Area of cross-section of the solenoid, A = $1.6 \times 10^{-4}\ m^2$

Current in the solenoid, I = 4 A

(a) The magnetic moment along the axis of the solenoid is calculated as:

M = nAI

= $2000 \times 1.6 \times 10^{-4} \times 4$

= 1.28 Am^2

(b) Magnetic field, B = 7.5×10^{-2} T

Angle between the magnetic field and the axis of the solenoid, θ = 30°

Torque, $\tau = MB\sin\theta$

$= 1.28 \times 7.5 \times 10^{-2} \sin 30°$

$= 4.8 \times 10^{-2}$ Nm

Since the magnetic field is uniform, the force on the solenoid is zero. The torque on the solenoid is 4.8×10^{-2} Nm.

Question 5.9:

A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}.

What is the moment of inertia of the coil about its axis of rotation?

Answer 5.9:

Number of turns in the circular coil, N = 16

Radius of the coil, r = 10 cm = 0.1 m

Cross-section of the coil, $A = \pi r^2 = \pi \times (0.1)^2\ m^2$

Current in the coil, I = 0.75 A

Magnetic field strength, $B = 5.0 \times 10^{-2}$ T

Frequency of oscillations of the coil, $\nu = 2.0\ s^{-1}$

∴ Magnetic moment, M = NIA $= NI\pi r^2$

$= 16 \times 0.75 \times \pi \times (0.1)^2$

$= 0.377\ J\ T^{-1}$

Frequency is given by the relation:

$$\nu = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$$

Where,

I = Moment of inertia of the coil

$$\therefore I = \frac{MB}{4\pi^2\nu^2}$$

$$= \frac{0.377 \times 5 \times 10^{-2}}{4\pi^2 \times (2)^2}$$

$$= 1.19 \times 10^{-4}\ \text{kg m}^2$$

Hence, the moment of inertia of the coil about its axis of rotation is $1.19 \times 10^{-4}\ \text{kg m}^2$.

Question 5.10:

A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22º with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.

Answer 5.10:

Horizontal component of earth's magnetic field, B_H = 0.35 G

Angle made by the needle with the horizontal plane = Angle of dip = $\delta = 22°$

Earth's magnetic field strength = B

We can relate B and B_H as:

$$B_H = B\cos\theta$$

$$\therefore B = \frac{B_H}{\cos\delta}$$

$$= \frac{0.35}{\cos 22°} = 0.377\ \text{G}$$

Hence, the strength of earth's magnetic field at the given location is 0.377 G.

Question 5.11:

At a certain location in Africa, a compass points 12º west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60º above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Answer 5.11:

Angle of declination, θ = 12°

Angle of dip, $\delta = 60°$

Horizontal component of earth's magnetic field, B_H = 0.16 G

Earth's magnetic field at the given location = B

We can relate B and B_H as:

$$B_H = B\cos\delta$$

$$\therefore B = \frac{B_H}{\cos\delta}$$

$$= \frac{0.16}{\cos 60°} = 0.32 \text{ G}$$

Earth's magnetic field lies in the vertical plane, 12° West of the geographic meridian, making an angle of 60° (upward) with the horizontal direction. Its magnitude is 0.32 G.

Question 5.12:

A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on **(a)** the axis, **(b)** the equatorial lines (normal bisector) of the magnet.

Answer 5.12:

Magnetic moment of the bar magnet, M = 0.48 J T^{-1}

(a) Distance, d = 10 cm = 0.1 m

The magnetic field at distance d, from the centre of the magnet on the axis is given by the relation:

$$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ Tm A^{-1}

$$\therefore B = \frac{4\pi \times 10^{-7} \times 2 \times 0.48}{4\pi \times (0.1)^3}$$

$$= 0.96 \times 10^{-4} \text{ T} = 0.96 \text{ G}$$

The magnetic field is along the S − N direction.

(b) The magnetic field at a distance of 10 cm (i.e., d = 0.1 m) on the equatorial line of the magnet is given as:

$$B = \frac{\mu_0 \times M}{4\pi \times d^3}$$

$$= \frac{4\pi \times 10^{-7} \times 0.48}{4\pi (0.1)^3}$$

$$= 0.48 \text{ G}$$

The magnetic field is along the N − S direction.

Question 5.13:

A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null−point (i.e., 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

Answer 5.13:

Earth's magnetic field at the given place, H = 0.36 G

The magnetic field at a distance d, on the axis of the magnet is given as:

$$B_1 = \frac{\mu_0}{4\pi} \frac{2M}{d^3} = H \qquad \dots (i)$$

Where,

μ_0 = Permeability of free space

M = Magnetic moment

The magnetic field at the same distance d, on the equatorial line of the magnet is given as:

$$B_2 = \frac{\mu_0 M}{4\pi d^3} = \frac{H}{2} \qquad \text{[Using equation } (i)\text{]}$$

Total magnetic field, $B = B_1 + B_2$

$$= H + \frac{H}{2}$$

$$= 0.36 + 0.18 = 0.54 \text{ G}$$

Hence, the magnetic field is 0.54 G in the direction of earth's magnetic field.

Question 5.14:

If the bar magnet in exercise 5.13 is turned around by 180º, where will the new null points be located?

Answer 5.14:

The magnetic field on the axis of the magnet at a distance d_1 = 14 cm, can be written as:

$$B_1 = \frac{\mu_0 2M}{4\pi (d_1)^3} = H \qquad \dots (1)$$

Where,

M = Magnetic moment

μ_0 = Permeability of free space

H = Horizontal component of the magnetic field at d_1

If the bar magnet is turned through 180°, then the neutral point will lie on the equatorial line.

Hence, the magnetic field at a distance d_2, on the equatorial line of the magnet can be written as:

$$B_2 = \frac{\mu_0 M}{4\pi (d_2)^3} = H \quad \dots (2)$$

Equating equations (1) and (2), we get:

$$\frac{2}{(d_1)^3} = \frac{1}{(d_2)^3}$$

$$\left(\frac{d_2}{d_1}\right)^3 = \frac{1}{2}$$

$$\therefore d_2 = d_1 \times \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

$$= 14 \times 0.794 = 11.1 \text{ cm}$$

The new null points will be located 11.1 cm on the normal bisector.

Question 5.15:

A short bar magnet of magnetic moment 5.25×10^{-2} J T^{-1} is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45º with earth's field on

(a) its normal bisector and **(b)** its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.

Answer 5.15:

Magnetic moment of the bar magnet, M = 5.25×10^{-2} J T^{-1}

Magnitude of earth's magnetic field at a place, H = 0.42 G = 0.42×10^{-4} T

(a) The magnetic field at a distance R from the centre of the magnet on the normal bisector is given by the relation:

$$B = \frac{\mu_0 M}{4\pi R^3}$$

Where,

μ_0 = Permeability of free space = 4π × 10^{-7} Tm A^{-1}

When the resultant field is inclined at 45° with earth's field, B = H

$$\therefore \frac{\mu_0 M}{4\pi R^3} = H = 0.42 \times 10^{-4}$$

$$R^3 = \frac{\mu_0 M}{0.42 \times 10^{-4} \times 4\pi}$$

$$= \frac{4\pi \times 10^{-7} \times 5.25 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}} = 12.5 \times 10^{-5}$$

$$\therefore R = 0.05 \text{ m} = 5 \text{ cm}$$

(b) The magnetic field at a distanced R' from the centre of the magnet on its axis is given as:

$$B' = \frac{\mu_0 2M}{4\pi R^3}$$

The resultant field is inclined at 45° with earth's field.

$$\therefore B' = H$$

$$\frac{\mu_0 2M}{4\pi (R')^3} = H$$

$$(R')^3 = \frac{\mu_0 2M}{4\pi \times H}$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times 5.25 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}} = 25 \times 10^{-5}$$

$$\therefore R' = 0.063 \text{ m} = 6.3 \text{ cm}$$