Exerise 1.1

1. *Use Euclid's division algorithm to find the HCF of*:

(*i*) *135 and 225* (*ii*) *196 and 38220*

(*iii*) *867 and 255*

Sol. (*i*) Applying Euclid's division algorithm on 135 and 225, to get

$$225 = 1 \times 135 + 90$$

Again, applying Euclid's division algorithm on 90 and 135, to get

$$135 = 1 \times 90 + 45$$

Again, applying Euclid's division algorithm on 45 and 90, to get

$$90 = 2 \times 45 + 0.$$

Now HCF (225, 135) = HCF (135, 90) = HCF (90, 45) = 45.

(*ii*) Applying Euclid's division algorithm on 196 and 38220, to get

$$38220 = 195 \times 196 + 0$$

$\therefore$ HCF (38220, 196) = 196.

(*iii*) Applying Euclid's division algorithm on 867 and 225, to get

$$867 = 3 \times 255 + 102$$

Again, applying Euclid's division algorithm on 102 and 255, to get

$$255 = 2 \times 102 + 51$$

Again, applying Euclid's division algorithm on 51 and 102, to get

$$102 = 2 \times 51 + 0$$

Now HCF (867, 255) = HCF (255, 102)

= HCF (102, 51) = 51.

2. *Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.*

Sol. Consider a, a positive integer, we apply division algorithm with q and $b = 6$, $0 \le r < 6$ such that

$$a = 6q + r$$

As $0 \le r < 6$. Therefore, possible remainders are 0, 1, 2, 3, 4, 5.

Number a can be $6q$, $6q + 1$, $6q + 2$, $6q + 3$, $6q + 4$ or $6q + 5$.

$6q = 2(3q) = 2m$; $6q + 1 = 2(3q) + 1 = 2m + 1$;

$6q + 2 = 2(3q + 1) = 2n$; $6q + 3 = 2(3q + 1) + 1 = 2n + 1$;

$6q + 4 = 2(3q + 2) = 2t$; $6q + 5 = 2(3q + 2) + 1 = 2t + 1$.

We note $6q$, $6q + 2$ and $6q + 4$ are of the form $2r$, $r \in$ N, which are even numbers; and $6q + 1$, $6q + 3$ and $6q + 5$ are of the form $2r + 1$, $r \in$ N, which are odd numbers.

Hence, $6q + 1$, $6q + 3$ and $6q + 5$ are positive and odd integers.

3. *An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?*

Sol. To find maximum number of columns, we have to find HCF of 616 and 32.

Using Euclid's division algorithm, we have

$$616 = 19 \times 32 + 8$$

Now applying Euclid's division algorithm on 32 and 8, we have

$$32 = 4 \times 8 + 0$$

$\therefore$ HCF (616, 32) = HCF (32, 8) = 8

Hence, maximum number of columns is 8.

4. *Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.*

[***Hint***: *Let x be any positive integer then it is of the form 3q, 3q + 1 or 3q + 2. Now square each of these and show that they can be rewritten in the form 3m or 3m + 1.*]

Sol. Consider positive integer x, with q and $b = 3$, $0 \le r < 3$

x can be of the form $3q$, $3q + 1$ and $3q + 2$ because

$r = 0, 1, 2$

There exist three cases:

Case I: $x = 3q$

$\Rightarrow \quad x^2 = (3q)^2 = 9q^2 = 3m,\ m = 3q^2$ ...(*i*)

Case II: $x = 3q + 1$

$\Rightarrow \quad x^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$

$= 3m + 1,\ m = 3q^2 + 2q$ ...(*ii*)

Case III: $x = 3q + 2$

$\Rightarrow \quad x^2 = (3q + 2)^2 = 9q^2 + 12q + 4$

$= 3(3q^2 + 4q + 1) + 1$

$= 3m + 1,\ m = 3q^2 + 4q + 1$ ...(*iii*)

Hence, from (*i*), (*ii*) and (*iii*), the square of any positive integer is either of the form $3m$ or $3m + 1$.

5. *Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.*

Sol. Consider positive integer a, with q and $b = 3$, $0 \le r < 3$.

a can be of the form $3q$, $3q + 1$ or $3q + 2$ because $r = 0, 1, 2$.

Consider $a = 3q$

$\Rightarrow \quad a^3 = 27q^3 = 9(3q^3) = 9m,\ m = 3q^3.$

Consider $a = 3q + 1$

$\Rightarrow \quad a^3 = (3q + 1)^3 = 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1 = 9m + 1,$

$m = 3q^3 + 3q^2 + q$

And consider $a = 3q + 2$

$\Rightarrow \quad a^3 = (3q + 2)^3 \quad = 27q^3 + 54q^2 + 36q + 8$

$= 9(3q^3 + 6q^2 + 4q) + 8 \quad = 9m + 8,$

$m = 3q^3 + 6q^2 + 4q$

Hence, cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

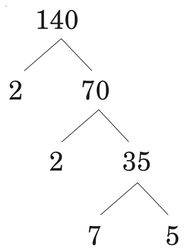
Exerise 1.2

1. *Express each number as a product of its prime factors:*

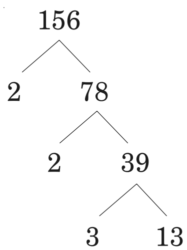
(*i*) *140* (*ii*) *156* (*iii*) *3825*

(*iv*) *5005* (*v*) *7429*

Sol. (*i*) $140 = 2 \times 2 \times 7 \times 5 = 2^2 \times 5 \times 7.$

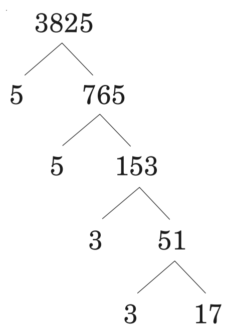


(*ii*) $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13.$

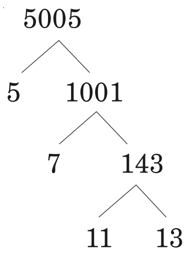


(*iii*) $3825 = 5 \times 5 \times 3 \times 3 \times 17$

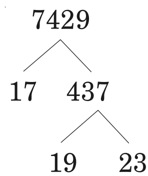
$= 5^2 \times 3^2 \times 17$



(*iv*) $5005 = 5 \times 7 \times 11 \times 13$.



(*v*) $7429 = 17 \times 19 \times 23$.



2. *Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.*

(*i*) *26 and 91* (*ii*) *510 and 92* (*iii*) *336 and 54*

Sol. (*i*) $26 = 2 \times 13$, $91 = 7 \times 13$

$\therefore$ HCF = 13 and LCM = $2 \times 7 \times 13 = 182$

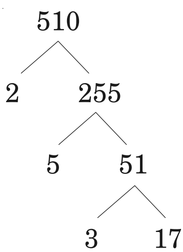
Now, HCF × LCM = $13 \times 182 = 2366$ ...(*i*)

Product of the numbers = $26 \times 91 = 2366$ ...(*ii*)

From (*i*) and (*ii*), we get the result.

(*ii*) $510 = 2 \times 5 \times 3 \times 17$ ...(*i*)

$92 = 2^2 \times 23$ ...(*ii*)



From (*i*) and (*ii*), we have

HCF = 2

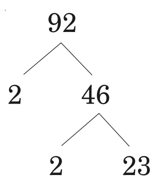
LCM = $2^2 \times 5 \times 3 \times 17 \times 23 = 23460$

Now, HCF × LCM

$= 2 \times 23460 = 46920$ ...(*iii*)

Product of the numbers = 510×92

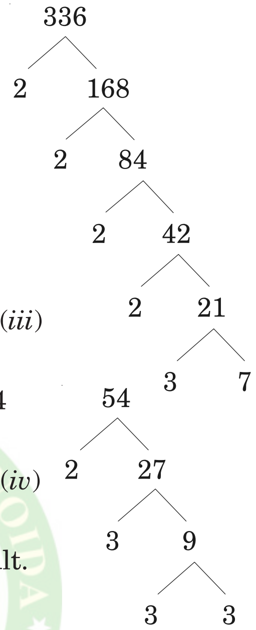
$= 46920$ ...(*iv*)



From (*iii*) and (*iv*), we get the result.

(*iii*) $336 = 2^4 \times 3 \times 7$ ...(*i*)

$54 = 2 \times 3^3$ ...(*ii*)



From (*i*) and (*ii*), we get

HCF $= 2 \times 3 = 6$

LCM $= 2^4 \times 3^3 \times 7 = 16 \times 27 \times 7$

$= 3024$

Now, HCF $\times$ LCM $= 6 \times 3024$

$= 18144$ ...(*iii*)

Product of the numbers $= 336 \times 54$

$= 18144$ ...(*iv*)

From (*iii*) and (*iv*), we get the result.

3. *Find the LCM and HCF of the following integers by applying the prime factorisation method.*

(*i*) *12, 15 and 21* (*ii*) *17, 23 and 29* (*iii*) *8, 9 and 25*

Sol. (*i*)

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

$\therefore$ HCF (12, 15, 21) = 3;

LCM (12, 15, 21) $= 2^2 \times 3 \times 5 \times 7 = 420$.

(*ii*)

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$\therefore$ HCF (17, 23, 29) = 1,

LCM (17, 23, 29) $= 1 \times 17 \times 23 \times 29 = 11339$.

(*iii*)

$8 = 2^3$

$9 = 3^2$

$25 = 5^2$

$\therefore$ HCF (8, 9, 25) = 1,

LCM (8, 9, 25) $= 2^3 \times 3^2 \times 5^2 = 1800$.

4. *Given that HCF (306, 657) = 9, find LCM (306, 657).*

Sol. We know for two numbers a and b HCF (a, b) × LCM $(a, b) = a \times b$

$$\therefore \quad \text{LCM}(306, 657) = \frac{306 \times 657}{\text{HCF}(306, 657)}$$

$$= \frac{306 \times 657}{9} = 22338.$$

5. *Check whether 6^n can end with the digit 0 for any natural number n.*

Sol. If 6^n for $n \in N$, ends with digit zero, then

$$6^n = 5 \times m + 0, \text{ for some } m \in N.$$

⇒ Prime factorisation of 6^n contains 5 as a prime factor.

Only prime factorisation of 6 is 3 and 2 and by Uniqueness of Fundamental Theorem of Arithmetic, 5 is not a prime factor.

Therefore, it contradicts our supposition that 5 is a factor of 6^n. Hence, 5 is not a factor of 6^n, *i.e.*, 6^n does not end with digit zero.

6. *Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.*

Sol. Composite numbers have more than two factors.

Number 7 × 11 × 13 + 13 = 13 (7 × 11 + 1) has factors 7 × 11 + 1, 1 and 13. As it has more than 2 factors it is a composite number, therefore, the number is a composite number.

Similarly, 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 = 5 (7 × 6 × 4 × 3 × 2 × 1 + 1)

⇒ The number has factors 1, 5 and (7 × 6 × 4 × 3 × 2 × 1 + 1).

As it has more than 2 factors, therefore, the number is a composite number.

7. *There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?*

Sol. If we have to find after how many minutes they again meet, we have to find the least common multiple of 18 and 12.

Now, $18 = 2 \times 3^2$ and $12 = 2^2 \times 3$

$\therefore$ LCM $= 2^2 \times 3^2 = 4 \times 9 = 36.$

Hence, after 36 minutes they will meet again at the starting point.

Exerise 1.3

1. *Prove that* $\sqrt{5}$ *is irrational.*

Sol. Let, if possible, $\sqrt{5}$ is a rational number.

$\therefore \quad \sqrt{5} = \frac{}{q}$, $p, q \in$ Z, $q \neq 0$; p and q are coprime.

$\Rightarrow \quad p^2 = 5q^2$ ...(*i*)

$\Rightarrow$ 5 divides p^2 $\Rightarrow$ 5 divides p.

Let $p = 5m$, $m \in$ Z. ...(*ii*)

$\therefore \quad (5m)^2 = 5q^2 \quad \Rightarrow \quad q^2 = 5m^2$ [From (*i*)]

$\Rightarrow$ 5 divides q^2 $\Rightarrow$ 5 divides q

$\therefore \quad q = 5n$, $n \in$ Z ...(*iii*)

From (*ii*) and (*iii*), we notice

p and q have 5 as common factor.

$\therefore$ p and q are not co-prime.

Hence, our supposition, $\sqrt{5}$ is rational, is wrong.

Hence, $\sqrt{5}$ is an irrational number.

2. *Prove that* $3 + 2\sqrt{5}$ *is irrational.*

Sol. Let if possible, $3 + 2\sqrt{5}$ is a rational number.

$\therefore \quad 3 + 2\sqrt{5} = \frac{a}{b}$, $a, b \in$ Z, $b \neq 0$; a and b are coprime.

$\Rightarrow \quad 2\sqrt{5} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$

$\Rightarrow \quad \sqrt{5} = \dfrac{a-3b}{2b}$ ...(*i*)

From (*i*), we notice R.H.S. is a rational number, using properties of rational numbers.

$\Rightarrow$ L.H.S., *i.e.,* $\sqrt{5}$ is also a rational number which is not true, as $\sqrt{5}$ is an irrational number. Hence, our supposition is wrong. Hence, $3 + 2\sqrt{5}$ is an irrational number.

3. *Prove that the following are irrationals*:

(*i*) $\dfrac{1}{\sqrt{2}}$ (*ii*) $7\sqrt{5}$ (*iii*) $6 + \sqrt{2}$

Sol. (*i*) Let, if possible, $\dfrac{1}{\sqrt{2}}$ is a rational number.

$\therefore \quad \dfrac{1}{\sqrt{2}} = \dfrac{a}{b}, a, b \in Z; a, b \neq 0.$

$\Rightarrow \quad \sqrt{2} = \dfrac{b}{a}$ ...(*i*)

From equation (*i*), we notice RHS is a rational number but LHS, *i.e.,* $\sqrt{2}$ is not rational. This contradicts the fact that $\sqrt{2}$ is an irrational. So, our supposition is wrong.

Hence, $\dfrac{1}{\sqrt{2}}$ is an irrational number.

(*ii*) Let, if possible, $7\sqrt{5}$ is a rational number.

$\therefore \quad 7\sqrt{5} = \dfrac{a}{b}, a, b \in Z; b \neq 0.$

$\Rightarrow \quad \sqrt{5} = \dfrac{a}{7b}$. Now LHS is an irrational number, whereas R.H.S. is a rational number. Therefore, LHS $\neq$ RHS So, our supposition is wrong.

Hence, $7\sqrt{5}$ is an irrational number.

(*iii*) Let, if possible, $6 + \sqrt{2}$ is a rational number.

$\therefore \quad 6 + \sqrt{2} = \frac{a}{b}, a, b \in Z, b \neq 0.$

$\Rightarrow \sqrt{2} = \frac{a}{b} - 6 = \frac{a - 6b}{b}$. Now LHS is an irrational number and RHS is a rational number using properties of rational numbers. Therefore, LHS $\neq$ RHS. So, our supposition is wrong.

Hence, $6 + \sqrt{2}$ is an irrational number.

Exerise 1.4

1. *Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:*

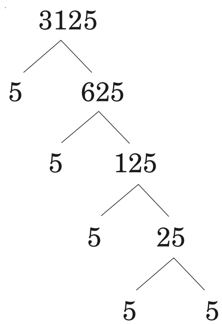
(i) $\frac{13}{125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{55}$

(iv) $\frac{15}{1600}$ (v) $\frac{9}{43}$ (vi) $\frac{3}{5^2}$

(vii) $\frac{129}{5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{5}{50}$

(x) $\frac{77}{10}$

Sol. (i) $\frac{13}{3125} = \frac{13}{5^5}$



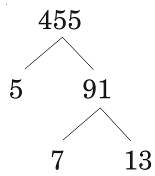
As denominator can be represented as power of prime factor 5. Hence, $\frac{13}{3125}$ has terminating decimal expansion.

(ii) $\frac{17}{8} = \frac{17}{2^3}$

As denominator has factors of the form $2^m\ 5^n$.

Hence, $\frac{17}{8}$ has terminating decimal expansion.

(*iii*) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$



As denominator cannot be represented in the form $2^m\ 5^n$, $m, n \in N$.

Hence, $\frac{64}{455}$ has non-terminating but repeating decimal expansion.

(*iv*) $\frac{15}{1600} = \frac{3}{320} = \frac{3}{2^6\ 5^1}$

As denominator has factors of the form $2^m\ 5^n$. Hence, $\frac{15}{1600}$ has terminating decimal expansion.

(*v*) $\frac{29}{343} = \frac{29}{7^3}$

As denominator has factors other than 2 and 5. Hence, $\frac{29}{343}$ has a non-terminating but repeating decimal expansion.

(*vi*) $\frac{23}{2^3\,5^2}$ is a terminating decimal expansion. As denominator is of the form $2^m 5^n$, $m, n \in$ N.

(*vii*) $\frac{129}{2^2 5^7 7^5}$ has a non-terminating but a repeating decimal expansion. As denominator has factors other than 2 and 5.

(*viii*) $\frac{6}{15} = \frac{2}{5}$

As denominator has factors of the term $2^m 5^n$. Hence, $\frac{6}{15}$ has terminating decimal expansion.

(ix) $\frac{35}{50} = \frac{7}{10} = \frac{7}{2 \times 5}$

As denominator has factors of the form $2^m 5^n$. Hence, $\frac{35}{50}$ has terminating decimal expansion.

(x) $\frac{77}{210} = \frac{77}{2 \times 3 \times 5 \times 7} = \frac{11}{2 \times 3 \times 5}$

As denominator has factors other than 2 and 5. Therefore, it has non-terminating but repeating decimal expansion.

2. *Write down the decimal expansions of those rational numbers in Question 1 above which have a terminating decimal expansions.*

Sol. (i) $\frac{13}{3125} = 0.00416$

(ii) $\frac{17}{8} = 2.125$

(iv) $\frac{15}{1600} = 0.009375$

(vi) $\frac{23}{2^3 5^2} = \frac{23}{200} = 0.115$

(viii) $\frac{6}{15} = 0.4$

(ix) $\frac{35}{50} = \frac{7}{10} = 0.7.$

3. *The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?*

(*i*) 43.123456789 (*ii*) 0.120120012000120000...

(*iii*) $43.\overline{123456789}$

Sol. (*i*) 43.123456789 is rational. It is terminating decimal, so it can be represented as $\frac{p}{q}$, $p, q \in Z$, $q \neq 0$ and denominator will be of form $2^m\ 5^n$, $m, n \in N$.

(*ii*) 0.120120012000... is not rational. It has a non-terminating, non-repeating decimal expansion. It cannot be represented in the form $\frac{p}{q}$.

(*iii*) $43.\overline{123456789}$ is rational. It has non-terminating but repeating decimal expansion. It can be represented in the form $\frac{p}{q}$, $q \neq 0$ but denominator cannot be represented in the form $2^m\ 5^n$, $m, n \in N$. Such denominator will have factors other than 2 and 5 also.