

Chapter- 5
Class 10 Arithmetic Progression

Exercise 5.1

1. *In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?*

(i) *The taxi fare after each km when the fare is ₹ 15 for the first km and ₹ 8 for each additional km.*

(ii) *The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.*

(iii) *The cost of digging a well after every metre of digging, when it costs ₹ 150 for the first metre and rises by ₹ 50 for each subsequent metre.*

(iv) *The amount of money in the account every year, when ₹ 10000 is deposited at compound interest at 8% per annum.*

Sol. (i) Yes. ₹ 15, ₹ 23, ₹ 31, is an A.P. as common difference is ₹ 8.

(ii) No. $1, \frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \dots$ is not an A.P. as common difference is not constant.

(iii) Yes. ₹ 150, ₹ 200, ₹ 250, is an A.P. as common difference is ₹ 50.

Chapter- 5
Class 10 Arithmetic Progression

(iv) No. ₹ 10000 $\times \frac{108}{100}$, ₹ 10000 $\times \left(\frac{108}{100}\right)^2$,

₹ 10000 $\left(\frac{108}{100}\right)^3$, is not an A.P. as common difference is not constant.

2. Write first four terms of the AP, when the first term a and the common difference d are given as follows:

(i) $a = 10$, $d = 10$ (ii) $a = -2$, $d = 0$

(iii) $a = 4$, $d = -3$ (iv) $a = -1$, $d = \frac{1}{2}$

(v) $a = -1.25$, $d = -0.25$

Sol. We will use: A.P. is $a, a + d, a + 2d, \dots$

(i) terms are 10, $10 + 10$, $10 + 20$, $10 + 30$, i.e., 10, 20, 30, 40.

(ii) terms are -2 , $-2 + 0$, $-2 + 0$, $-2 + 0$, i.e., -2 , -2 , -2 , -2 .

(iii) terms are 4, $4 - 3$, $4 - 6$, $4 - 9$, i.e., 4, 1, -2 , -5 .

(iv) terms are -1 , $-1 + \frac{1}{2}$, $-1 + 1$, $-1 + \frac{3}{2}$,

i.e., -1 , $-\frac{1}{2}$, 0, $\frac{1}{2}$.

Chapter- 5

Class 10 Arithmetic Progression

(v) terms are $-1.25, -1.25 - 0.25, -1.25 - 0.50, -1.25 - 0.75$, i.e., $-1.25, -1.50, -1.75, -2.00$.

3. For the following APs, write the first term and the common difference:

(i) $3, 1, -1, -3, \dots$ (ii) $-5, -1, 3, 7, \dots$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$ (iv) $0.6, 1.7, 2.8, 3.9, \dots$

Sol. (i) $a = 3, d = 1 - 3 = -2$

(ii) $a = -5, d = -1 + 5 = 4$

(iii) $a = \frac{1}{3}, d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) $a = 0.6, d = 1.7 - 0.6 = 1.1$.

4. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) $2, 4, 8, 16, \dots$ (ii) $2, \frac{5}{2}, 3, \frac{7}{2}, \dots$

(iii) $-1.2, -3.2, -5.2, -7.2, \dots$

(iv) $-10, -6, -2, 2, \dots$

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

(vi) $0.2, 0.22, 0.222, 0.2222, \dots$

(vii) $0, -4, -8, -12, \dots$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$ (ix) $1, 3, 9, 27, \dots$

(x) $a, 2a, 3a, 4a, \dots$ (xi) a, a^2, a^3, a^4, \dots

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \dots$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

(xiv) $1^2, 3^2, 5^2, 7^2, \dots$ (xv) $1^2, 5^2, 7^2, 73, \dots$

Sol. (i) $4 - 2 = 2; 8 - 4 = 4; 16 - 8 = 8$.

Difference is not constant. Hence, it is not an A.P.

(ii) $\frac{5}{2} - 2 = \frac{1}{2}; 3 - \frac{5}{2} = \frac{1}{2}; \frac{7}{2} - 3 = \frac{1}{2}$.

Difference is constant, i.e., $d = \frac{1}{2}$. Hence, it is an A.P.

Next three terms are:

Chapter- 5

Class 10 Arithmetic Progression

$$\frac{7}{2} + \frac{1}{2} = 4, 4 + \frac{1}{2} = \frac{9}{2} \text{ and } \frac{9}{2} + \frac{1}{2} = 5.$$

(iii) $-3.2 + 1.2 = -2; -5.2 + 3.2 = -2; -7.2 + 5.2 = -2.$

Difference is constant, i.e., $d = -2$. Hence, an A.P.

Next three terms are:

$$-7.2 - 2 = -9.2, -9.2 - 2 = -11.2$$

$$\text{and } -11.2 - 2 = -13.2.$$

(iv) $-10, -6, -2, 2, \dots$

Here, $-6 + 10 = 4, -2 + 6 = 4, 2 + 2 = 4 \dots$

Difference is constant, i.e., $d = 4$.

Hence, it is an A.P.

Three more terms are:

$$2 + 4 = 6, 6 + 4 = 10 \text{ and } 10 + 4 = 14.$$

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

Here, $3 + \sqrt{2} - 3$

$$= \sqrt{2}, 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2},$$

$$3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2} \dots$$

Difference is constant, i.e.,

$$d = \sqrt{2}. \text{ Hence, it is an A.P.}$$

Next three terms are:

$$3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2},$$

$$3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2} \text{ and}$$

$$3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}.$$

(vi) $0.22 - 0.2 = 0.02; 0.222 - 0.22 = 0.002.$

Difference is not constant. Hence, it is not an A.P.

(vii) $0, -4, -8, -12, \dots$

Here, $-4 - 0 = -4, -8 + 4 = -4,$

$$-12 + 8 = -4, \dots$$

Difference is constant, i.e.,

$$d = -4. \text{ Hence, it is an A.P.}$$

Chapter- 5

Class 10 Arithmetic Progression

Next three terms are:

$$-12 - 4 = -16, -16 - 4 = -20$$

$$\text{and } -20 - 4 = -24.$$

$$(viii) -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$$

$$\text{Here, } -\frac{1}{2} + \frac{1}{2} = 0, -\frac{1}{2} + \frac{1}{2} = 0, -\frac{1}{2} + \frac{1}{2} = 0.$$

Difference is constant, *i.e.*, $d = 0$. Hence, it is an A.P.

Next three terms are:

$$-\frac{1}{2} + 0 = -\frac{1}{2}, -\frac{1}{2} + 0 = -\frac{1}{2} \text{ and } -\frac{1}{2} + 0 = -\frac{1}{2}.$$

$$(ix) 1, 3, 9, 27, \dots$$

$$\text{Here, } 3 - 1 = 2, 9 - 3 = 6, 27 - 9 = 18, \dots$$

Difference is not constant. Hence, it is not an A.P.

$$(x) 2a - a = a; 3a - 2a = a; 4a - 3a = a.$$

Difference is constant, *i.e.*, $d = a$. Hence, it is an A.P.

Next three terms are: $4a + a = 5a$, $5a + a = 6a$ and $6a + a = 7a$.

$$(xi) a^2 - a = a(a - 1); a^3 - a^2 = a^2(a - 1).$$

Difference is not constant. Hence, it is not an A.P.

$$(xii) \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \dots$$

$$2\sqrt{2} - \sqrt{2} = \sqrt{2}; 3\sqrt{2} - 2\sqrt{2} = \sqrt{2};$$

$$4\sqrt{2} - 3\sqrt{2} = \sqrt{2}.$$

Difference is constant, *i.e.*, $d = \sqrt{2}$. Hence,

it is an A.P.

$$\text{Next three terms are: } 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50},$$

$$5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$$

$$\text{and } 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}.$$

$$(xiii) \sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$$

Chapter- 5

Class 10 Arithmetic Progression

Here, $\sqrt{6} - \sqrt{3} \neq \sqrt{9} - \sqrt{6} \neq \sqrt{12} - \sqrt{6}$.

Difference is not constant. Hence, it is not an A.P.

(xiv) $1^2, 3^2, 5^2, 7^2, \dots, \text{i.e., } 1, 9, 25, 49, \dots$

Here, $9 - 1 = 8, 25 - 9 = 16, 49 - 25 = 24, \dots$

Difference is not constant. Hence, it is not an A.P.

(xv) $1^2, 5^2, 7^2, 73, \dots, \text{i.e., } 1, 25, 49, 73, \dots$

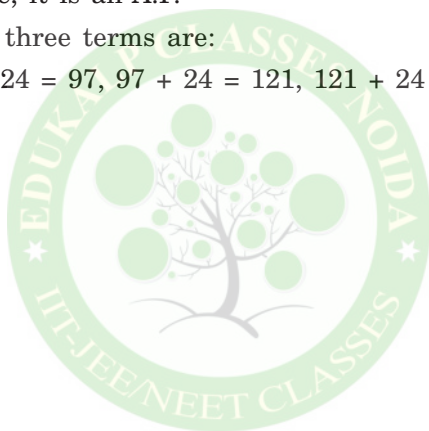
Here, $25 - 1 = 24, 49 - 25 = 24, 73 - 49 = 24, \dots$

Difference is constant, *i.e.*, $d = 24$.

Hence, it is an A.P.

Next three terms are:

$73 + 24 = 97, 97 + 24 = 121, 121 + 24 = 145$.



Chapter- 5

Class 10 Arithmetic Progression

Exercise 5.2

1. Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n th term of the AP:

	a	d	n	a_n
(i)	7	3	8
(ii)	- 18	10	0
(iii)	- 3	18	- 5
(iv)	- 18.9	2.5	3.6
(v)	3.5	0	105

- Sol.**
- (i) $a_n = a + (n - 1)d = 7 + (8 - 1) 3 = 7 + 21 = 28.$
- (ii) $a_n = a + (n - 1)d \Rightarrow 0 = - 18 + (10 - 1)d$
 $\Rightarrow 9d = 18 \Rightarrow d = 2.$
- (iii) $a_n = a + (n - 1)d \Rightarrow - 5 = a + (18 - 1)(- 3)$
 $\Rightarrow a = - 5 + 51 = 46.$
- (iv) $3.6 = - 18.9 + (n - 1)(2.5)$
 $\Rightarrow 3.6 + 18.9 = (n - 1)(2.5)$
 $\Rightarrow 22.5 = 2.5n - 2.5 \Rightarrow 2.5n = 25$
 $\Rightarrow n = 10.$
- (v) $a_n = a + (n - 1)d = 3.5 + (105 - 1) 0 = 3.5.$

Chapter- 5

Class 10 Arithmetic Progression

2. Choose the correct choice in the following and justify:

(i) 30th term of the AP: 10, 7, 4,, is

(A) 97 (B) 77 (C) - 77 (D) - 87

(ii) 11th term of the AP: $-3, -\frac{1}{2}, 2, \dots$, is

(A) 28 (B) 22 (C) - 38 (D) $-48\frac{1}{2}$

Sol. (i) $a = 10, d = 7 - 10 = -3$,
 $a_{30} = 10 + (30 - 1)(-3) = 10 - 87 = -77$.
Hence, option (C) is correct.

(ii) $a = -3, d = -\frac{1}{2} + 3 = \frac{5}{2}$;

$$a_{11} = -3 + (11 - 1)\frac{5}{2} = -3 + 25 = 22.$$

Hence, option (B) is correct.

3. In the following APs, find the missing terms in the boxes:

(i) 2, , 26

(ii) , 13, , 3

(iii) 5, , , $9\frac{1}{2}$ (iv) -4, , , , , 6

(v) , 38, , , , -22

Sol. (i) $a_1 = 2, a_3 = 26; \quad 26 = 2 + (3 - 1)d$
 $\Rightarrow 24 = 2d \Rightarrow d = 12$.
 $\therefore a_2 = 2 + 12 = 14$.

Hence, 14.

(ii) $a_2 = 13 \Rightarrow a + d = 13$ and $a_4 = 3 \Rightarrow a + 3d = 3$
Solving for a and d , we get $a = 18, d = -5$
 $\therefore a_3 = 18 + 2 \times (-5) = 8$.

Hence, 18, 8.

(iii) $a_1 = 5, a_4 = \frac{19}{2}; \quad \frac{19}{2} = 5 + (4 - 1)d$
 $\Rightarrow 3d = \frac{9}{2} \Rightarrow d = \frac{3}{2}$

Chapter- 5

Class 10 Arithmetic Progression

$$\therefore a_2 = 5 + \frac{3}{2} = \frac{13}{2} = 6\frac{1}{2}; a_3 = \frac{13}{2} + \frac{3}{2} = \frac{16}{2} = 8.$$

Hence, $\boxed{6\frac{1}{2}}, \boxed{8}$

(iv) $a = -4$ and $a_6 = 6 \Rightarrow -4 + 5d = 6$

$$\Rightarrow d = 2$$

$$\therefore a_2 = -2, a_3 = 0, a_4 = 2, a_5 = 4.$$

Hence, $\boxed{-2}, \boxed{0}, \boxed{2}, \boxed{4}$.

(v) **Hint:** $a + d = 38; a + 5d = -22$.

On solving, we get $a = 53, d = -15$. Find a_3, a_4, a_5 .

4. Which term of the AP: 3, 8, 13, 18,, is 78?

Sol. Let $a_n = 78$, then $78 = 3 + (n - 1)5 \Rightarrow n = 16$.

5. Find the number of terms in each of the following APs:

(i) 7, 13, 19,, 205 (ii) $18, 15\frac{1}{2}, 13, \dots, -47$

Sol. (i) Let $a_n = 205, a = 7, d = 6$

$$\text{Then, } 205 = 7 + (n - 1)6 \Rightarrow 198 = (n - 1)6$$

$$\Rightarrow n - 1 = 33 \Rightarrow n = 34.$$

(ii) Let $a_n = -47, a = 18, d = -\frac{5}{2}$.

$$\text{Then, } -47 = 18 + (n - 1)\left(-\frac{5}{2}\right) \Rightarrow -65 = (n - 1)\left(-\frac{5}{2}\right)$$

$$\Rightarrow n - 1 = 26 \Rightarrow n = 27.$$

6. Check whether -150 is a term of the AP: 11, 8, 5, 2,

Sol. Let $a_n = -150$, then $-150 = 11 + (n - 1)(-3)$

$$\Rightarrow -161 = -3n + 3 \Rightarrow 3n = 164$$

$$\Rightarrow n = \frac{164}{3} = 54.67 \notin \mathbb{N}.$$

Hence, -150 is not a term of the given AP.

7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.

Sol. $a + 10d = 38; a + 15d = 73 \Rightarrow a = -32, d = 7$.

$$\therefore a_{31} = -32 + 30 \times 7 = -32 + 210 = 178.$$

Chapter- 5

Class 10 Arithmetic Progression

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Sol. $a_3 = 12$ and $a_{50} = 106 \Rightarrow a + 2d = 12$

$$\text{and } a + 49d = 106$$

Solving for a and d , we get, $a = 8, d = 2$

$$\therefore a_{29} = 8 + (29 - 1) \times 2 = 8 + 56 = 64.$$

9. If the 3rd and the 9th terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Sol. $a + 2d = 4; a + 8d = -8 \Rightarrow a = 8, d = -2.$

$$\text{Let } a_n = 0 \Rightarrow 8 + (n - 1)(-2) = 0 \Rightarrow n = 5.$$

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Sol. $a_{17} = a_{10} + 7 \Rightarrow a + 16d = a + 9d + 7 \Rightarrow d = 1.$

11. Which term of the AP: 3, 15, 27, 39, will be 132 more than its 54th term?

Sol. Let n th term of given AP is 132 more than its 54th term.

$$\text{Here, } a = 3, d = 12$$

$$\therefore a_n = a_{54} + 132$$

$$\therefore 3 + (n - 1)12 = 3 + (54 - 1)12 + 132$$

$$\Rightarrow (n - 1)12 = 768 \Rightarrow n - 1 = 64 \Rightarrow n = 65$$

Hence, 65th term of the AP is 132 more than its 54th term.

12. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms?

Sol. Let common difference of two APs be d . $\{a_n\}$ and $\{a'_n\}$ are two APs:

$$a_{100} - a'_{100} = 100 \Rightarrow (a_1 + 99d) - (a'_1 + 99d) = 100$$

$$\Rightarrow a_1 - a'_1 = 100. \quad \dots(i)$$

$$\text{Now, } a_{1000} - a'_{1000} = (a_1 + 999d) - (a'_1 + 999d)$$

$$= a_1 - a'_1 = 100 \dots \quad [\text{From (i)}]$$

13. How many three-digit numbers are divisible by 7?

Sol. Three-digit numbers divisible by 7 are 105, 112,, 994.

Chapter- 5

Class 10 Arithmetic Progression

Let the required number of numbers be n

$$\therefore 994 = 105 + (n - 1)7 \Rightarrow 7n = 896 \Rightarrow n = 128.$$

14. *How many multiples of 4 lie between 10 and 250?*

Sol. Multiples of 4 between 10 and 250 are 12, 16,, 248.

Here, $a = 12$, $d = 4$, let $a_n = 248$

$$\therefore 248 = 12 + (n - 1)4 \Rightarrow (n - 1)4 = 236$$

$$\Rightarrow n - 1 = 59 \Rightarrow n = 1 + 59 \Rightarrow n = 60.$$

15. *For what value of n , are the n th terms of two APs: 63, 65, 67, and 3, 10, 17, equal?*

Sol. $63 + (n - 1)2 = 3 + (n - 1)7$

$$\Rightarrow 60 = 5(n - 1) \Rightarrow n = 13.$$

16. *Determine the AP whose third term is 16 and 7th term exceeds the 5th term by 12.*

Sol. $a_3 = 16$ and $a_7 = a_5 + 12$

$$\Rightarrow a + 2d = 16 \text{ and } a + 6d = a + 4d + 12$$

$$\Rightarrow d = 6 \text{ and } a = 4.$$

Hence, A.P. is 4, 10, 16, 22,

17. *Find the 20th term from the last term of the AP:*

$$3, 8, 13,, 253.$$

Sol. From the end, $a = 253$, $d = -5$

$$\therefore \text{20th term from the end} = 253 + (20 - 1)(-5) \\ = 253 - 95 = 158.$$

18. *The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.*

Sol. $(a + 3d) + (a + 7d) = 24 \Rightarrow 2a + 10d = 24$

$$\Rightarrow a + 5d = 12 \quad \dots(i)$$

$$\text{and } (a + 5d) + (a + 9d) = 44 \Rightarrow 2a + 14d = 44$$

$$\Rightarrow a + 7d = 22 \quad \dots(ii)$$

Solving (i) and (ii), we get

$$a = -13, d = 5$$

Thus, first three terms are -13 , -8 and -3 .

19. *Subba Rao started work in 1995 at an annual salary of ₹ 5000 and received an increment of ₹ 200 each year. In which year did his income reach ₹ 7000?*

Chapter- 5

Class 10 Arithmetic Progression

Sol. Let $a_n = 7000$. Then $7000 = 5000 + (n - 1)200$

$$\Rightarrow 2000 = (n - 1) \cdot 200 \Rightarrow n = 11.$$

In the year 2006, i.e., 11th year, his income reaches ₹ 7000.

- 20.** *Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the n th week, her weekly savings become ₹ 20.75, find n .*

Sol. $a_n = 20.75$. Then $20.75 = 5 + (n - 1)(1.75)$

$$\Rightarrow 15.75 = (n - 1)(1.75) \Rightarrow n - 1 = 9 \Rightarrow n = 10$$

In 10th week, her savings will be ₹ 19.75.



Chapter- 5
Class 10 Arithmetic Progression

Exercise 5.3

1. Find the sum of the following APs:

(i) 2, 7, 12,, to 10 terms.

(ii) - 37, - 33, - 29,, to 12 terms.

(iii) 0.6, 1.7, 2.8,, to 100 terms.

(iv) $\frac{1}{15}$, $\frac{1}{12}$, $\frac{1}{10}$,, to 11 terms.

Sol. (i) $a = 2, d = 5, n = 10.$

$$\therefore S_{10} = \frac{10}{2} [4 + (10 - 1)5] = 5[49] = 245.$$

(ii) $a = - 37, d = 4, n = 12$

$$\therefore S_{12} = \frac{12}{2} [- 74 + 11 \times 4] = 12 \times (- 15) = - 180.$$

(iii) $a = 0.6, d = 1.1, n = 100$

$$\begin{aligned}\therefore S_{100} &= \frac{100}{2} [1.2 + (100 - 1)(1.1)] \\ &= 50 \times 110.1 = 5505.\end{aligned}$$

(iv) $a = \frac{1}{15}, d = \frac{1}{12} - \frac{1}{15} = \frac{1}{60}, n = 11$

$$\begin{aligned}\therefore S_{11} &= \frac{11}{2} \left[\frac{2}{15} + (11 - 1) \frac{1}{60} \right] = \frac{11}{2} \left[\frac{2}{15} + \frac{1}{6} \right] \\ &= \frac{11}{2} \times \frac{3}{10} = \frac{33}{20}.\end{aligned}$$

Chapter- 5

Class 10 Arithmetic Progression

2. Find the sums given below:

- (i) $7 + 10\frac{1}{2} + 14 + \dots + 84$
- (ii) $34 + 32 + 30 + \dots + 10$
- (iii) $-5 + (-8) + (-11) + \dots + (-230)$.

Sol. (i) $a = 7, d = \frac{21}{2} - 7 = \frac{7}{2}, l = 84$.

$$\therefore 84 = 7 + (n - 1)\frac{7}{2} \Rightarrow 77 = (n - 1)\frac{7}{2}$$
$$\Rightarrow n = 23.$$

$$\therefore S_{23} = \frac{23}{2}(7 + 84) = \frac{23}{2} \times 91 = \frac{2093}{2} = 1046\frac{1}{2}.$$

(ii) $a = 34, d = -2, a_n = 10$

$$\Rightarrow 10 = 34 + (n - 1)(-2)$$

$$\Rightarrow -24 = (n - 1)(-2) \Rightarrow n = 13$$

$$\therefore S_{13} = \frac{13}{2}[34 + 10] = \frac{13}{2} \times 44 = 13 \times 22 = 286.$$

(iii) $a = -5, d = -3, l = -230$

$$-230 = -5 + (n - 1)(-3) \Rightarrow -225 = (n - 1)(-3)$$

$$\Rightarrow n = 76$$

$$\therefore S_{76} = \frac{76}{2}(-5 - 230) = 38 \times (-235) = -8930.$$

3. In an AP,

- (i) given $a = 5, d = 3, a_n = 50$, find n and S_n .
- (ii) given $a = 7, a_{13} = 35$, find d and S_{13} .
- (iii) given $a_{12} = 37, d = 3$, find a and S_{12} .
- (iv) given $a_3 = 15, S_{10} = 125$, find d and a_{10} .
- (v) given $d = 5, S_9 = 75$, find a and a_9 .
- (vi) given $a = 2, d = 8, S_n = 90$, find n and a_n .
- (vii) given $a = 8, a_n = 62, S_n = 210$, find n and d .
- (viii) given $a_n = 4, d = 2, S_n = -14$, find n and a .
- (ix) given $a = 3, n = 8, S = 192$, find d .
- (x) given $l = 28, S = 144$, and there are total 9 terms.
Find a .

Chapter- 5

Class 10 Arithmetic Progression

Sol. (i) $a_n = 50 \Rightarrow 5 + (n - 1)3 = 50$
 $\Rightarrow (n - 1)3 = 45 \Rightarrow n - 1 = 15$
 $\Rightarrow n = 16$

$$S_n = S_{16} = \frac{16}{2}(10 + 15 \times 3) = 8 \times 55 = 440.$$

(ii) $35 = 7 + 12d \Rightarrow d = \frac{7}{3}$

$$\therefore S_{13} = \frac{13}{2} \left[14 + 12 \times \frac{7}{3} \right] = \frac{13}{2} \times 42$$
$$= 13 \times 21 = 273.$$

(iii) $a_{12} = 37 \Rightarrow a + 33 = 37$
 $\Rightarrow a = 4$

$$S_{12} = \frac{12}{2} [8 + (12 - 1)3] = 6 \times 41 = 246.$$

(iv) $a + 2d = 15$ and $125 = \frac{10}{2}[2a + 9d]$

$$\Rightarrow a + 2d = 15 \text{ and } 25 = 2a + 9d$$

Solving, we get $a = 17, d = -1$

$$\therefore a_{10} = a + 9d = 17 - 9 = 8.$$

(v) $S_9 = 75 \Rightarrow 75 = \frac{9}{2}[2a + (9 - 1)5]$

$$\Rightarrow a = -\frac{35}{3}, a_9 = -\frac{35}{3} + (9 - 1)5$$

$$= -\frac{35}{3} + 40 = \frac{85}{3}.$$

(vi) $S_n = 90 \Rightarrow 90 = \frac{n}{2}[4 + (n - 1)8].$

$$\Rightarrow 90 = \frac{n}{2}[8n - 4] \Rightarrow 90 = 4n^2 - 2n$$

$$\Rightarrow 2n^2 - n - 45 = 0 \Rightarrow 2n^2 - 10n + 9n - 45 = 0$$

$$\Rightarrow 2n(n - 5) + 9(n - 5) = 0$$

$$\Rightarrow (2n + 9)(n - 5) = 0$$

Chapter- 5

Class 10 Arithmetic Progression

$$\Rightarrow 2n + 9 = 0 \text{ or } n - 5 = 0$$

$$\Rightarrow n = \frac{-9}{2} \text{ (rejected) or } n = 5.$$

$$\therefore a_n = a_5 = 2 + 4 \times 8 = 34.$$

$$(vii) \quad 210 = \frac{n}{2}(a + a_n) \Rightarrow 210 = \frac{n}{2}(8 + 62)$$

$$\Rightarrow n = 6$$

$$a_6 = 62 \Rightarrow 62 = 8 + (6 - 1) \times d$$

$$\Rightarrow 54 = 5d \Rightarrow d = \frac{54}{5}.$$

$$(viii) \quad a_n = 4 \Rightarrow a + (n - 1)d = 4$$

$$\Rightarrow a + (n - 1) \cdot 2 = 4 \quad \dots(i)$$

$$S_n = -14 \Rightarrow -14 = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow -14 = \frac{n}{2}[8 - 4(n - 1) + 2(n - 1)] \quad [\text{Using (i)}]$$

$$\Rightarrow -14 = \frac{n}{2}[10 - 2n] \Rightarrow -14 = n(5 - n)$$

$$\Rightarrow n^2 - 5n - 14 = 0 \Rightarrow (n - 7)(n + 2) = 0$$

$$\Rightarrow n - 7 = 0 \text{ or } n + 2 = 0$$

$$\Rightarrow n = 7, -2 \text{ (rejected)}$$

Substituting the value of $n = 7$ in (i), we get

$$a + 12 = 4 \Rightarrow a = -8.$$

$$(ix) \quad S_8 = 192 \Rightarrow 192 = \frac{8}{2}[6 + (8 - 1)d]$$

$$\Rightarrow 192 = 4[6 + 7d] \Rightarrow 48 = 6 + 7d$$

$$\Rightarrow d = 6.$$

$$(x) \quad S_9 = \frac{9}{2}(a + l) \Rightarrow 144 = \frac{9}{2}(a + 28)$$

$$\Rightarrow a + 28 = 32 \Rightarrow a = 4.$$

4. How many terms of the AP: 9, 17, 25, must be taken to give a sum of 636?

Sol. Let $S_n = 636 \Rightarrow 636 = \frac{n}{2}[18 + (n - 1)8]$

$$\Rightarrow 636 = n[9 + 4n - 4] \Rightarrow 4n^2 + 5n - 636 = 0$$

$$\Rightarrow 4n^2 + 53n - 48n - 636 = 0$$

Chapter- 5

Class 10 Arithmetic Progression

$$\Rightarrow n(4n + 53) - 12(4n + 53) = 0$$

$$\Rightarrow (4n + 53)(n - 12) = 0$$

$$\Rightarrow n = -\frac{53}{4} \text{ (rejected) or } n = 12.$$

5. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Sol. $a_1 = 5, a_n = 45, S_n = 400$

$$\therefore 400 = \frac{n}{2}[5 + 45] \Rightarrow n = 16$$

$$\therefore a_{16} = 45 \Rightarrow 5 + 15d = 45 \Rightarrow d = \frac{40}{15} = \frac{8}{3}.$$

6. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Sol. $a = 17, a_n = 350, d = 9$

$$\therefore 350 = 17 + (n - 1)9 \Rightarrow n = 38.$$

$$\therefore S_{38} = \frac{38}{2}(17 + 350) = 19 \times 367 = 6973.$$

7. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Sol. $a_{22} = 149 \Rightarrow 149 = a + 21 \times 7 \Rightarrow a = 2$

$$\therefore S_{22} = \frac{22}{2}[4 + (22 - 1) \times 7] = 11 \times 151 = 1661.$$

8. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Sol. $a_2 = 14$ and $a_3 = 18 \Rightarrow a + d = 14$

$$\text{and } a + 2d = 18$$

$$\Rightarrow a = 10, d = 4$$

$$\therefore S_{51} = \frac{51}{2}[20 + (51 - 1) \times 4] = 51 \times 110 = 5610.$$

9. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Sol. $\frac{7}{2}[2a + (7 - 1)d] = 49 \Rightarrow a + 3d = 7 \quad \dots(i)$

Chapter- 5

Class 10 Arithmetic Progression

and $\frac{17}{2}[2a + (17 - 1)d] = 289 \Rightarrow a + 8d = 17 \quad \dots(ii)$

Solving (i) and (ii), we get

$$a = 1, d = 2$$

$$\therefore S_n = \frac{n}{2}[2 + (n - 1)2] = \frac{n}{2} \times 2n = n^2.$$

10. Show that $a_1, a_2, \dots, a_n, \dots$ form an AP where a_n is defined as below:

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Sol. (i) $a_n = 3 + 4n$

$$\begin{aligned} d &= a_n - a_{n-1} = (3 + 4n) - \{3 + 4(n - 1)\} \\ &= 3 + 4n - 3 - 4n + 4 = 4 \end{aligned}$$

As d is free from n , i.e., d is constant. Hence $\{a_n\}$ is an A.P.

$$a_1 = 7$$

$$\therefore S_{15} = \frac{15}{2}[14 + (15 - 1)4] = \frac{15}{2} \times 70 = 525.$$

(ii) $a_n = 9 - 5n,$

$$d = (9 - 5n) - \{9 - 5(n - 1)\} = -5$$

As d is free from n , i.e., d is constant. Hence $\{a_n\}$ is an A.P.

Further $a_1 = 4.$

$$\therefore S_{15} = \frac{15}{2}[8 + 14 \times (-5)] = \frac{15}{2} \times (-62) = -465.$$

11. If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n th terms.

Sol. $S_n = 4n - n^2 \quad \dots(i)$

$$S_1 = 4 - 1 = 3 = a$$

$$S_2 = 8 - 4 = 4 \quad \text{[From (i)]}$$

$$a_n = S_n - S_{n-1} = (4n - n^2) - \{4(n - 1) - (n - 1)^2\}$$

$$= 4n - n^2 - 4n + 4 + n^2 - 2n + 1$$

$$\Rightarrow a_n = 5 - 2n \Rightarrow a_2 = 1, a_3 = -1 \text{ and } a_{10} = -15.$$

Chapter- 5

Class 10 Arithmetic Progression

12. Find the sum of the first 40 positive integers divisible by 6.

Sol. 6, 12, 18, 40 terms

$$S_{40} = \frac{40}{2} [12 + (40 - 1)6] = 20 \times 246 = 4920.$$

13. Find the sum of the first 15 multiples of 8.

Sol. 8 + 16 + 24 + 15 terms.

$$S_{15} = \frac{15}{2} [16 + (15 - 1)8] = \frac{15}{2} \times 128 = 960.$$

14. Find the sum of the odd numbers between 0 and 50.

Sol. Numbers between 0 and 50 are 1, 3, 5, 49.

Here, $n = 25$, $a = 1$, $d = 2$

$$\therefore S_{25} = \frac{25}{2} [2 + (25 - 1)2] = \frac{25}{2} \times 50 = 625.$$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: ₹ 200 for the first day, ₹ 250 for the second day, ₹ 300 for the third day, etc., the penalty for each succeeding day being ₹ 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Sol. 200 + 250 + 300 + 30 terms

$$S_{30} = \frac{30}{2} [400 + (30 - 1)50] = 15 \times 1850 = ₹ 27750.$$

16. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

Sol. Let prizes be a , $a - 20$, $a - 40$; $a - 60$, 7 times.

The sum of these prizes is ₹ 700.

$$\therefore \frac{7}{2} [2a + (7 - 1)(-20)] = 700 \Rightarrow a - 60 = 100$$

$$\Rightarrow a = 160.$$

The prizes are of ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will

Chapter- 5

Class 10 Arithmetic Progression

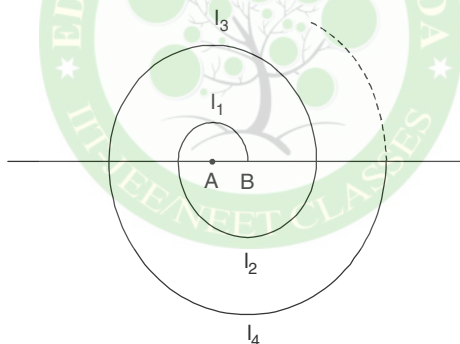
plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Sol. Total number of trees planted by the students.

$$= 3 + 6 + 9 + \dots + 36 = \frac{12}{2} [3 + 36] = 6 \times 39 = 234.$$

18. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, as shown in figure What is the total length of such a spiral made up of thirteen consecutive semicircles. $\left(\text{Take } \pi = \frac{22}{7} \right)$

[**Hint.** Length of successive semicircle is $l_1, l_2, l_3, l_4, \dots$ with centres at A, B, A, B, respectively.]



Sol. Length of semicircle with radius 0.5 cm = 0.5π cm

Length of semicircle with radius 1.0 cm = π cm

Length of semicircle with radius 1.5 cm = 1.5π cm

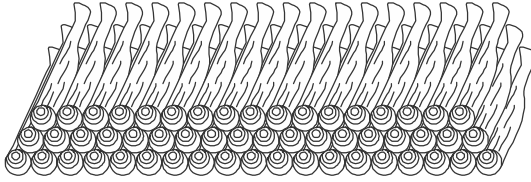
These form an A.P.; which is $0.5\pi, \pi, 1.5 \pi, \dots$

$$\begin{aligned} \therefore S_{13} &= \frac{13}{2} [\pi + (13 - 1)0.5\pi] = \frac{13}{2} \times 7\pi \\ &= \frac{13}{2} \times 7 \times \frac{22}{7} = 143 \text{ cm.} \end{aligned}$$

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see figure). In how many rows are the 200 logs placed and how many logs are in the top row?

Chapter- 5

Class 10 Arithmetic Progression



Sol. $20 + 19 + 18 + \dots n \text{ terms} = 200$

$$\frac{n}{2} [40 + (n - 1)(-1)] = 200.$$

$$\Rightarrow \frac{n}{2} [41 - n] = 200 \Rightarrow 41n - n^2 = 400$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow (n - 25)(n - 16) = 0$$

$$\Rightarrow n = 25, 16$$

$$S_{25} = \frac{25}{2} [40 + 24 \times (-1)]$$

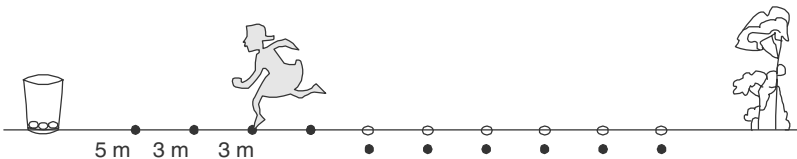
$$= \frac{25}{2} \times 16 = 200;$$

$$S_{16} = \frac{16}{2} [40 + 15 \times (-1)] = 8 \times 25 = 200$$

$n = 25$ is ruled out as $a_{25} = -4 < 0$.

$\therefore n = 16$. And $a_{16} = 20 + (16 - 1) \cdot (-1) = 5$.

- 20.** In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see fig.).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

Chapter- 5

Class 10 Arithmetic Progression

[**Hint.** To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is

$$2 \times 5 + 2 \times (5 + 3)]$$

Sol. To pick up first, second, third, potatoes, distances covered are 10 m, 16 m, 22 m, These form an A.P. with $a = 10$, $d = 6$.

$$\therefore S_{10} = \frac{10}{2} [20 + (10 - 1)6] = 5 \times 74 = 370 \text{ m.}$$



Chapter- 5
Class 10 Arithmetic Progression

Exercise 5.4

1. Which term of the AP: 121, 117, 113,, is its first negative term?

[Hint. Find n for $a_n < 0$]

Sol. AP is 121, 117, 113,

Let $a_n < 0 \Rightarrow 121 + (n - 1)(-4) < 0$

$\Rightarrow 125 - 4n < 0 \Rightarrow 125 < 4n$

$\Rightarrow n > 31\frac{1}{4}$

$\Rightarrow n = 32,$

i.e., 32nd term is the first negative term.

2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of first sixteen terms of the AP.

Sol. $a_3 + a_7 = 6 \Rightarrow a + 2d + a + 6d = 6$

$\Rightarrow 2a + 8d = 6 \Rightarrow a + 4d = 3 \quad \dots(i)$

Also $(a + 2d)(a + 6d) = 8$

$\Rightarrow (3 - 4d + 2d)(3 - 4d + 6d) = 8 \quad \text{[From (i)]}$

$\Rightarrow (3 - 2d)(3 + 2d) = 8 \Rightarrow 9 - 4d^2 = 8$

$\Rightarrow 4d^2 = 1 \Rightarrow d = \pm \frac{1}{2}$

When $d = \frac{1}{2}, a = 1, S_{16} = \frac{16}{2} \left[2 + (16-1)\frac{1}{2} \right]$

$= 8 \times \frac{19}{2} = 76$

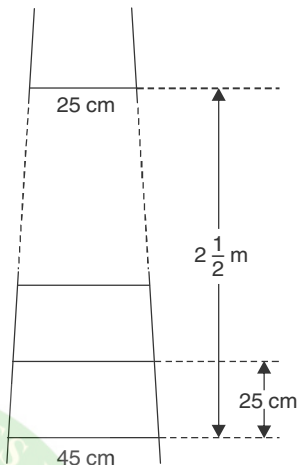
When $d = -\frac{1}{2}, a = 5, S_{16} = \frac{16}{2} \left[10 + (16-1)\left(-\frac{1}{2}\right) \right]$

Chapter- 5

Class 10 Arithmetic Progression

$$= 8 \times \frac{5}{2} = 20.$$

3. A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



[**Hint:** Number of rungs = $\frac{250}{25} + 1$]

Sol. Number of rungs = $\frac{2\frac{1}{2} \text{ m}}{25 \text{ cm}} + 1$
 $= \frac{250 \text{ cm}}{25 \text{ cm}} + 1 = 11$

$$a_1 = 45 \text{ and } a_{11} = 25 \Rightarrow 45 + 10d = 25$$

$$\Rightarrow d = \frac{-20}{10} = -2$$

$$\therefore S_{11} = \frac{11}{2} [90 + 10 \times (-2)]$$

$$= 5.5 \times 70 \text{ cm} = 385 \text{ cm}.$$

4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x .

[**Hint:** $S_{x-1} = S_{49} - S_x$]

Sol. House numbers are: 1, 2, 3,, x ,, 49

According to the given conditions,

$$S_{x-1} = S_{49} - S_x$$

$$\Rightarrow \frac{x-1}{2} [2 + (x-2)1] = \frac{49}{2} [2 + 48] - \frac{x}{2} [2 + (x-1)1]$$

$$\Rightarrow (x-1)(x) = 49 \times 50 - x(x+1)$$

Chapter- 5

Class 10 Arithmetic Progression

$$\Rightarrow x^2 - x = 2450 - x^2 - x \Rightarrow x^2 = 1225$$

$$\Rightarrow x = 35.$$

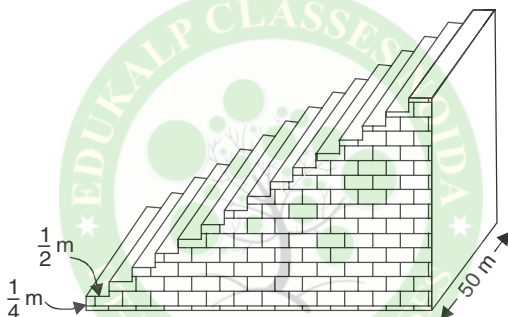
5. A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure).

Calculate the total volume of concrete required to build the terrace.

[Hint: Volume of concrete required to build the first step

$$= \frac{1}{4} \times \frac{1}{2} \times 50 \text{ m}^3]$$



Sol. Volume for the first step = $\frac{1}{4} \times \frac{1}{2} \times 50 \text{ m}^3$

Volume for the second step = $\frac{2}{4} \times \frac{1}{2} \times 50 \text{ m}^3$

Volume for the third step = $\frac{3}{4} \times \frac{1}{2} \times 50 \text{ m}^3$

These form an A.P. which are

$\frac{1}{4} \times \frac{1}{2} \times 50, \frac{2}{4} \times \frac{1}{2} \times 50, \frac{3}{4} \times \frac{1}{2} \times 50 \dots \dots$ upto 15 terms

Here, $a = \frac{50}{8}$ and $d = \frac{100}{8} - \frac{50}{8} = \frac{50}{8}$

$\therefore S_{15} = \frac{15}{2} \left[\frac{100}{8} + 14 \times \frac{50}{8} \right] = \frac{15}{2} \times 100 = 750.$

Hence, total volume of concrete required is 750 m^3 .