Exerise 6.1

1. *Fill in the blanks using the correct word given in brackets*:

(*i*) *All circles are* ________ . (*congruent, similar*)

(*ii*) *All squares are* ________ . (*similar, congruent*)

(*iii*) *All* ________ *triangles are similar.* (*isosceles, equilateral*)

(*iv*) *Two polygons of the same number of sides are similar, if* (*a*) *their corresponding angles are* ________ *and* (*b*) *their corresponding sides are* ________ . (*equal, proportional*)

Sol. (*i*) similar (*ii*) similar

(*iii*) equilateral (*iv*) equal, proportional.

2. *Give two different examples of pair of*

(*i*) *similar figures* (*ii*) *non-similar figures.*

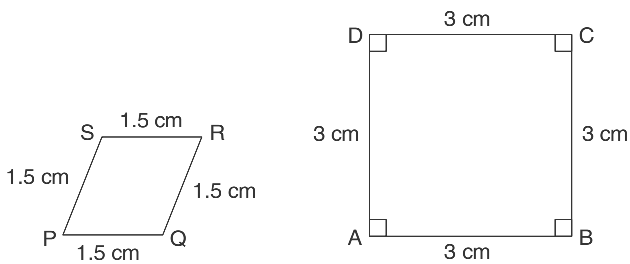
Sol. (*i*) (*a*) A passport size photograph and a postcard size photograph of the same person from the same negative.

(*b*) Equilateral triangles.

(*ii*) (*a*) A square and a rhombus.

(*b*) A circular dinning table and a rectangular dinning table.

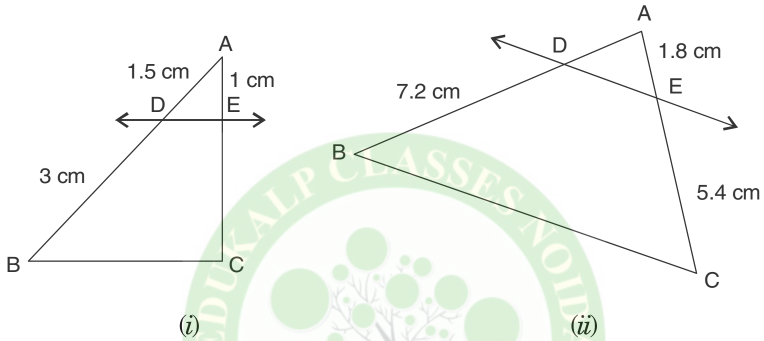
3. *State whether the following quadrilaterals are similar or not*:



Sol. Not similar as corresponding angles are not equal.

Exerise 6.2

1. *In figure, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).*



Sol. In figure (*i*), $\frac{AD}{BD} = \frac{AE}{EC} \Rightarrow \frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2$ cm.

In figure (*ii*), $\frac{AD}{BD} = \frac{AE}{EC} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4} \Rightarrow AD = 2.4$ cm.

2. *E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR:*

(*i*) *PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm*

(*ii*) *PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm*

(*iii*) *PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm.*

Sol. (*i*) $\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$

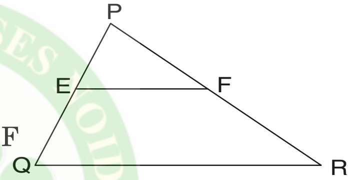
$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{3}{2} = 1.5$

Here, $\frac{PE}{EQ} \neq \frac{PF}{FR}$, so EF is not parallel to QR.

(*ii*) $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$ and $\frac{PF}{RF} = \frac{8}{9}$

Here, $\frac{PE}{EQ} = \frac{PF}{RF}$,

so by inverse of Basic Proportionality Theorem, EF is parallel to QR.

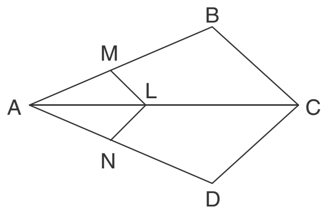


(*iii*) $\frac{PQ}{PE} = \frac{1.28}{0.18} = \frac{64}{9}$ and $\frac{PR}{PF} = \frac{2.56}{0.36} = \frac{64}{9}$

As $\frac{PQ}{PE} = \frac{PR}{PF}$, so EF is parallel to QR.

3. *In figure, if LM ∥ CB and LN ∥ CD,*

prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Sol. Given: In the given figure, LM ∥ BC and LN ∥ CD.

To prove: $\frac{AM}{AB} = \frac{AN}{AD}$.

Proof: In ΔABC, LM || BC

$\Rightarrow \frac{AM}{MB} = \frac{AL}{LC}$ ...(*i*) [Basic Proportionality theorem]

In ΔACD, LN || CD

$\Rightarrow \frac{AN}{ND} = \frac{AL}{LC}$ ...(*ii*) [Basic Proportionality theorem]

From (*i*) and (*ii*), we get

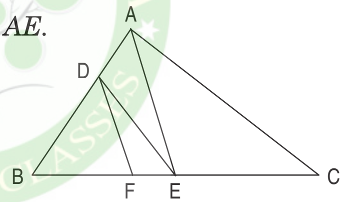
$\Rightarrow \frac{AM}{MB} = \frac{AN}{ND} \Rightarrow \frac{MB}{AM} = \frac{ND}{AN}$

$\Rightarrow \frac{MB}{AM} + 1 = \frac{ND}{AN} + 1$

$\Rightarrow \frac{AB}{AM} = \frac{AD}{AN} \Rightarrow \frac{AM}{AB} = \frac{AN}{AD}$. **Hence proved.**

4. *In figure, DE || AC and DF || AE.*

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Sol. Proof: In ΔABC, DE || AC

$\Rightarrow \frac{BE}{EC} = \frac{BD}{AD}$ ...(*i*) [Basic Proportionality theorem]

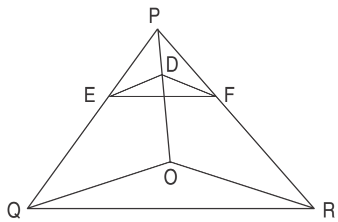
In ΔABE, DF || AE

$\Rightarrow \frac{BD}{DA} = \frac{BF}{FE}$ ...(*ii*) [Reason same as above]

From (*i*) and (*ii*), we get $\frac{BE}{EC} = \frac{BF}{FE}$. **Hence proved.**

5. *In figure, DE || OQ and DF || OR.*

Show that EF || QR.



Sol. Proof: In ΔPQO, DE || OQ.

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ ...(*i*) [Basic Proportionality Theorem]

In ΔPOR, DF || OR

$$\frac{PD}{DO} = \frac{PF}{FR} \quad ...(ii) \text{ [Reason same as above]}$$

From (*i*) and (*ii*), we get

$$\frac{PE}{EQ} = \frac{PF}{FR} \quad ...(iii)$$

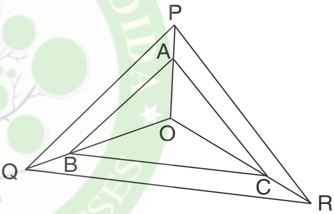
In ΔPQR, $\frac{PE}{EQ} = \frac{PF}{FR}$ [From (*iii*)]

⇒ EF || QR.

[Using converse of Basic Proportionality Theorem]

Hence proved.

6. *In figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.*



Sol. Proof: In ΔPQO, AB || PQ

$$\Rightarrow \frac{PA}{AO} = \frac{QB}{BO} \quad ...(i) \text{ [Basic Proportionality Theorem]}$$

In ΔPOR, AC || PR

$$\Rightarrow \frac{PA}{AO} = \frac{RC}{CO} \quad ...(ii) \text{ [Reason same as above]}$$

From (*i*) and (*ii*), we have

$$\frac{QB}{BO} = \frac{RC}{CO} \quad ...(iii)$$

In ΔQOR, $\frac{QB}{BO} = \frac{RC}{CO}$ [From (*iii*)]

⇒ BC || QR.

[Using converse of Basic Proportionality Theorem]

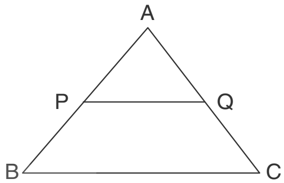
7. *Using Basic Proportionality Theorem, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).*

Sol. **Given:** In a ΔABC, P is mid-point of AB, *i.e.,* AP = PB and PQ $\parallel$ BC.

To prove: Q is mid-point of AC.

Proof: In ΔABC, PQ $\parallel$ BC

$$\Rightarrow \quad \frac{AP}{PB} = \frac{AQ}{QC} \qquad ...(i) \text{ [Basic Proportionality Theorem]}$$



Also, AP = PB ...(*ii*) [Given]

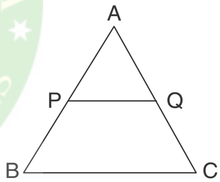
From (*i*) and (*ii*),

$$\Rightarrow \quad 1 = \frac{AQ}{QC}$$

$\Rightarrow$ AQ = QC $\Rightarrow$ Q is mid-point of AC.

8. *Using Converse of Basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).*

Sol. **Given:** In a ΔABC, P and Q are mid-points of sides AB and AC respectively.



To prove: PQ $\parallel$ BC.

Proof: As P and Q are mid-points of AB and AC.

$\therefore$ AP = PB and AQ = QC ...(*i*)

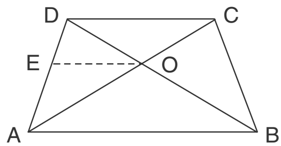
$$\Rightarrow \quad \frac{AP}{PB} = \frac{AQ}{QC} \qquad ...(ii) \text{ [From } (i)]$$

$$\text{In } \Delta ABC, \quad \frac{AP}{PB} = \frac{AQ}{QC} \qquad \text{[From } (ii)]$$

$\Rightarrow$ PQ $\parallel$ BC.

[Using converse of basic Proportionality Theorem]

9. *ABCD is a trapezium in which AB $\parallel$ DC and its diagonals intersect each other at the point O. Show that* $\frac{AO}{BO} = \frac{CO}{DO}$.



Sol. Given: A trapezium ABCD, AB ∥ CD.
Diagonals AC and BD intersect at O.

To prove: $\frac{AO}{BO} = \frac{CO}{DO}$.

Construction: Draw OE ∥ AB, meeting AD at E.
Proof: In ΔADB, OE ∥ AB.

$\Rightarrow \quad \frac{DE}{EA} = \frac{DO}{BO}$ ...(*i*) [Basic Proportionality Theorem]

As AB ∥ OE and AB ∥ DC
∴ OE ∥ DC
In ΔADC, OE ∥ DC

$\Rightarrow \quad \frac{DE}{EA} = \frac{CO}{AO}$ ...(*ii*) [Basic Proportionality Theorem]

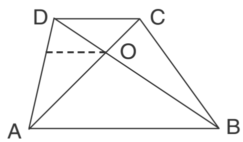
From (*i*) and (*ii*), we have

$\frac{CO}{AO} = \frac{DO}{BO} \Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$. **Hence proved.**

10. *The diagonals of a quadrilateral ABCD intersect each other at the point O such that* $\frac{AO}{BO} = \frac{CO}{DO}$. *Show that ABCD is a trapezium.*

Sol. Given: In quadrilateral ABCD, O is the point of intersection AC and BD such that

$$\frac{AO}{BO} = \frac{CO}{DO}$$



To prove: ABCD is a trapezium.
Construction: Draw OE ∥ AB
Proof: In ΔDAB, OE ∥ AB

$\frac{OB}{OD} = \frac{AE}{ED}$ ...(*i*) [Basic Proportionality Theorem]

Also $\frac{OA}{OC} = \frac{OB}{OD}$ [Given] $\Rightarrow \frac{OA}{OC} = \frac{AE}{ED}$ [From (*i*)]

In ΔADC, $\frac{OA}{OC} = \frac{AE}{ED}$

$\Rightarrow$ OE $\parallel$ DC ...(*ii*)

[Converse of Basic Proportionality Theorem]

Also OE $\parallel$ AB. ...(*iii*) [From construction]

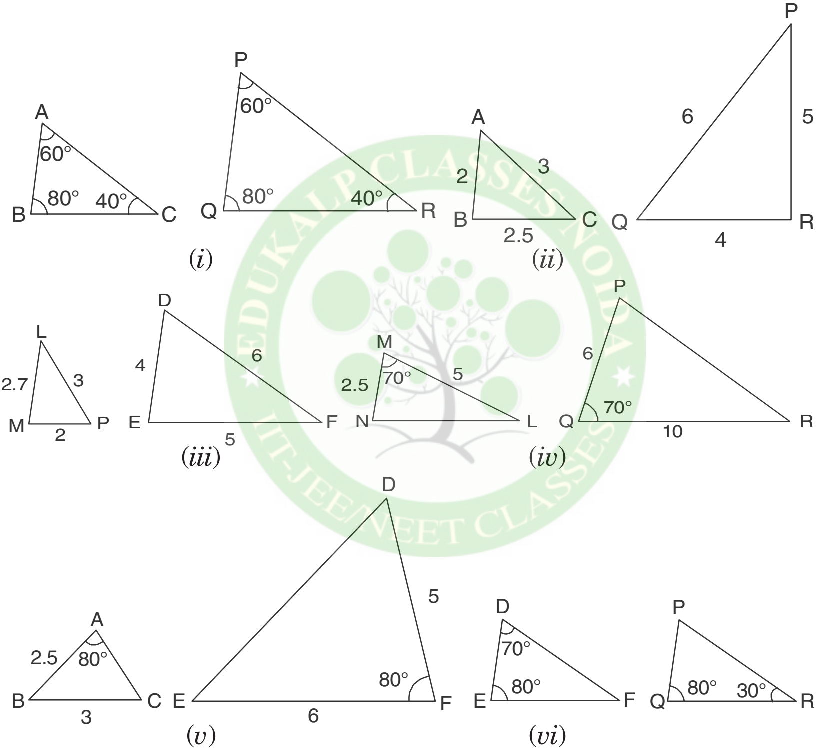
From (*ii*) and (*iii*), we have

DC $\parallel$ AB

$\therefore$ Quadrilateral ABCD is a trapezium.

Exerise 6.3

1. *State which pairs of triangles in figure are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:*



Sol. (*i*) $\Delta ABC \sim \Delta PQR$. [AAA similarity]

(*ii*) $\dfrac{AB}{QR} = \dfrac{BC}{PR} = \dfrac{AC}{PQ} = \dfrac{1}{2}$

$\Rightarrow \quad \Delta ABC \sim \Delta QRP.$ [SSS similarity]

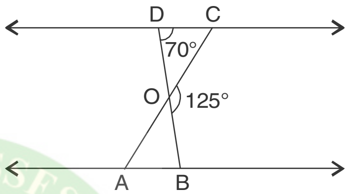
(*iii*) Not similar, as sides are not in proportional.

(*iv*) Not similar, SAS not satisfied.

(*v*) Not similar.

(*vi*) Similar as $\Delta DEF \sim \Delta PQR$. [AAA similarity]

2. *In figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 125°$ and $\Delta CDO = 70°$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.*



Sol. $\Delta ODC \sim \Delta OBA$

$\Rightarrow \quad \angle OAB = \angle OCD$ ...(*i*)

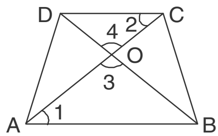
$\angle DOC = 180° - 125° = 55°$

$\angle OCD = 180° - (70° + 55°) = 55°$

And $\angle OAB = 55°$ [From (*i*)]

3. *Diagonals AC and BD of a trapezium ABCD with AB ∥ DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.*

Sol. Consider triangles AOB and COD.



In these triangles,

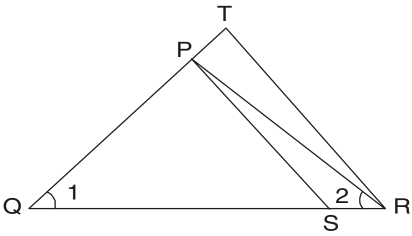
$\angle 1 = \angle 2$ [Alternate angles]

$\angle 4 = \angle 3$ [Vertically opposite angles]

$\therefore \quad \Delta AOB \sim \Delta COD$ [AA similarity]

$\therefore \quad \frac{OA}{OC} = \frac{OB}{OD}.$ **Hence proved.**

4. *In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.*



Sol. In ΔPQR, $\angle 1 = \angle 2 \Rightarrow PQ = PR$ ...(*i*)

Given, $\frac{QR}{QS} = \frac{QT}{PR} \Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ [Using (*i*)]

In triangles PQS and QTR,

$$\frac{QR}{QS} = \frac{QT}{PQ}$$ [Proved above]

and $\angle 1$ is common.

$\therefore$ $\Delta PQS \sim \Delta TQR$. [SAS similarity]

5. *S and T are points on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.*

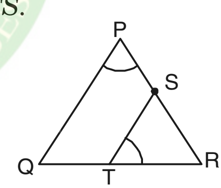
Sol. In triangles PQR and TRS,

$\angle P = \angle RTS$ [Given]

and $\angle R$ is common.

$\therefore$ $\Delta RPQ \sim \Delta RTS$

[AA similarity]



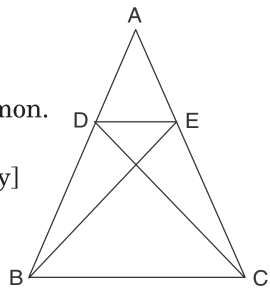
6. *In figure, if $\Delta ABE \cong \Delta ACD$, show that $\Delta ADE \sim \Delta ABC$.*

Sol. $\Delta ABE \cong \Delta ACD$

$\Rightarrow$ AB = AC and AE = AD

$\Rightarrow$ $\frac{AB}{AD} = \frac{AC}{AE}$ and $\angle A$ is common.

$\Rightarrow$ $\Delta ADE \sim \Delta ABC$ [SAS similarity]



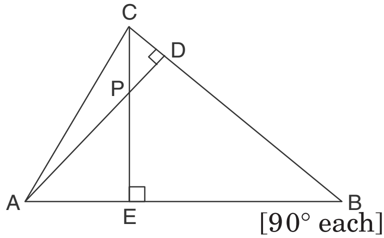
7. *In figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:*

(i) $\Delta\ AEP \sim \Delta\ CDP$

(ii) $\Delta\ ABD \sim \Delta\ CBE$

(iii) $\Delta\ AEP \sim \Delta\ ADB$

(iv) $\Delta\ PDC \sim \Delta\ BEC$



Sol. (i) In ΔAEP and ΔCDP,

$\angle AEP = \angle CDP$ [90° each]

$\angle APE = \angle CPD$ [Vertically opposite angles]

$\therefore$ $\Delta AEP \sim \Delta CDP$ [AA similarity]

(ii) In ΔABD and ΔCBE,

$\angle ADB = \angle CEB$ [90° each]

and ∠B is common.

$\therefore$ $\Delta ABD \sim \Delta CBE$. [AA similarity]

(iii) Let $\angle DAB = x$ ...(i)

$\Rightarrow$ $\angle PAE = x$ ...(ii)

In ΔAPE and ΔADB,

$\angle PAE = \angle DAB$ [From (i) and (ii)]

$\angle AEP = \angle ADB$ [90° each]

$\therefore$ $\Delta AEP \sim \Delta ADB$ [AA similarity]

(iv) In ΔPDC and ΔBEC,

∠C is common and $\angle CDP = \angle CEB$ [90° each]

$\therefore$ $\Delta PDC \sim \Delta BEC$. [AA similarity]

Hence proved.

8. *E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB.*

Sol. Consider ΔABE and ΔCFB

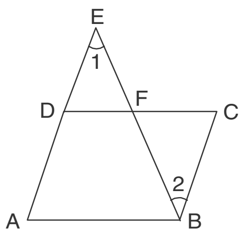
$\angle A = \angle C$

[Opposite angles of a parallelogram]

$\angle 1 = \angle 2$ [Alternate angles]

$\therefore$ $\Delta ABE \sim \Delta CFB$ [AA similarity]

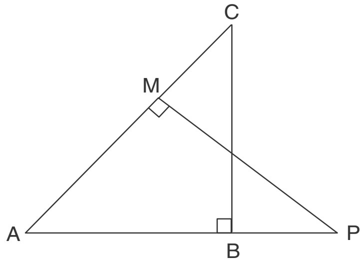
Hence proved.



9. *In figure, ABC and AMP are two right triangles, right angled at B and M respectively. Prove that:*

(*i*) $\Delta ABC \sim \Delta AMP$

(*ii*) $\frac{CA}{PA} = \frac{BC}{MP}$



Sol. In ΔABC and ΔAMP,
∠A is common
and ∠ABC = ∠AMP. [90° each]

∴ (*i*) ΔABC ~ ΔAMP [AA similarity]

(*ii*) $\frac{CA}{PA} = \frac{BC}{MP}$. [Corresponding sides]

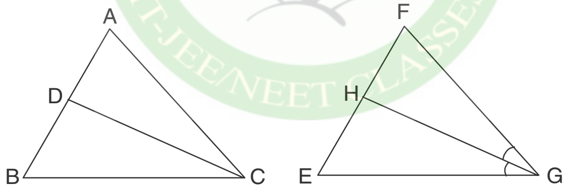
Hence proved.

10. *CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, show that*:

(*i*) $\frac{CD}{GH} = \frac{AC}{FG}$ (*ii*) ΔDCB ~ ΔHGE

(*iii*) ΔDCA ~ ΔHGF

Sol. (*i*)



ΔABC ~ ΔFEG

∴ ∠A = ∠F ...(*i*)

and ∠ACB = ∠FGE

$\Rightarrow \frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$

$\Rightarrow \angle ACD = \angle FGH$ ...(*ii*)

[∵ CD and GH are the bisectors of ∠ACB and ∠FGE respectively]

∴ ΔACD ~ ΔFGH [AA similarity] [Using (*i*), (*ii*)]

$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$.

(*ii*) Consider ΔDCB and ΔHGE

$\angle B = \angle E$ ...(*iii*) [$\because$ $\Delta ABC \sim \Delta FEG$]

$\angle ACB = \angle FGE$ [$\because$ $\Delta ABC \sim \Delta FEG$]

$$\frac{1}{2}\angle ACB = \frac{1}{2}\angle FGE$$

$\Rightarrow$ $\angle DCB = \angle HGE$ ...(*iv*) [$\because$ CD and GH are the bisectors of $\angle ACB$ and $\angle FGE$ respectively]

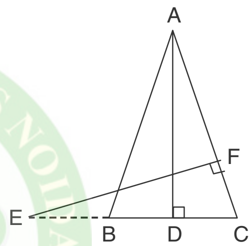
$\therefore$ $\Delta DCB \sim \Delta HGE$

...[AA similarity] [From (*iii*) and (*iv*)]

(*iii*) Refer result (*i*). $\Delta ACD \sim \Delta FGH$

$\Rightarrow$ $\Delta DCA \sim \Delta HGF$.

11. *In figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF.*



Sol. Consider ΔABD and ΔECF,

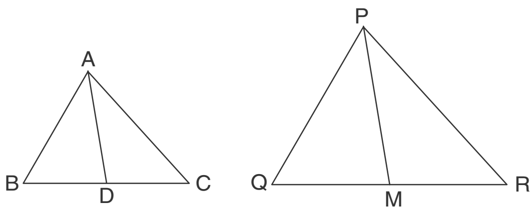
$\angle ADB = \angle EFC$ [90° each]

$\angle ABD = \angle C$ [$\because$ AB = AC]

$\therefore$ $\Delta ABD \sim \Delta ECF$. [AA similarity]

Hence proved.

12. *Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR* (*see figure*). *Show that ΔABC ~ ΔPQR.*



Sol. $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$ ...(*i*) [Given]

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PB}$$

...(*ii*) $\left(\because \quad BD = \frac{1}{2}BC \text{ and } QM = \frac{1}{2}QR\right)$

$\therefore$ $\Delta ABD \sim \Delta PQM$ [From (*ii*) SSS similarity]

$\Rightarrow$ $\angle B = \angle Q$ ...(*iii*)

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{BC}{QR}$$ [From (*i*)]

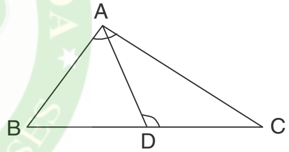
and $\angle B = \angle Q$ [From (*iii*)]

$\Rightarrow$ $\Delta ABC \sim \Delta PQR$. [SAS similarity]

Hence proved.

13. *D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.*

Sol. Consider ΔABC and ΔADC



$\angle BAC = \angle ADC$ [Given]

$\angle C$ is common.

$\therefore$ $\Delta ABC \sim \Delta DAC$ [AA similarity]

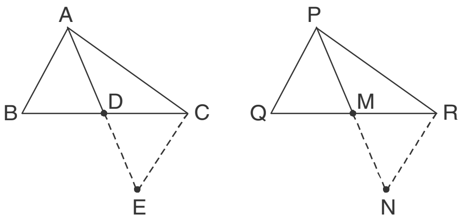
$\therefore$ $\frac{CA}{CD} = \frac{CB}{CA} \Rightarrow CA^2 = CB \cdot CD$. **Hence proved.**

14. *Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$.*

Sol. Given: $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$.

To prove: $\Delta ABC \sim \Delta PQR$.

Construction: Produce AD and PM to E and N respectively such that AD = DE and PM = MN. Join EC and RN.



Proof: Consider ΔABD and ΔCDE

$AD = DE$ [Construction]

$BD = DC$ [D is mid-point of BC]

and $\angle ADB = \angle CDE$ [Vertically opposite angles]

$\therefore$ $\Delta ABD \cong \Delta ECD$ [SAS]

$\therefore$ $AB = CE$ ...(*i*)

Similarly, we can show that $PQ = RN$ ...(*ii*)

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$ [Given]

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{2AD}{2PM}$$ [From (*i*) and (*ii*)]

$$\Rightarrow \frac{CE}{RN} = \frac{AC}{PR} = \frac{AE}{PN}$$ [Using construction]

$\Rightarrow$ $\Delta AEC \sim \Delta PNR$ [SSS similarity]

$\therefore$ $\angle DAC = \angle MPR$ ...(*iii*)

Similarly, we can show that

$\angle BAD = \angle QPM$ ...(*iv*)

$\Rightarrow$ $\angle BAD + \angle DAC = \angle QPM + \angle MPR$ [From (*iii*), (*iv*)]

$\Rightarrow$ $\angle BAC = \angle QPR$ ...(*v*)

In ΔABC and ΔPQR,

$$\frac{AB}{PQ} = \frac{AC}{PR}$$ ...(*vi*) [Given]

And $\angle BAC = \angle QPR$ [From (*v*)]

$\Rightarrow$ $\Delta ABC \sim \Delta PQR$ [SAS similarity]

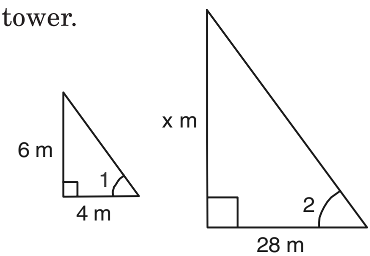
Hence proved.

15. *A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.*

Sol. Let x m be the height of the tower.

Two triangles are similar as at the same time $\angle 1 = \angle 2$ (see figure)

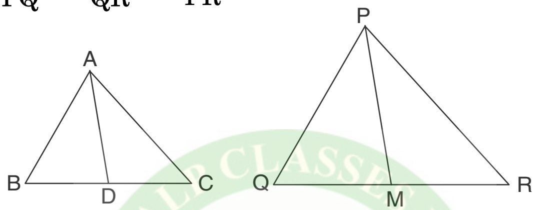
$$\therefore \frac{6}{4} = \frac{x}{28} \Rightarrow x = 42 \text{ m.}$$



16. *If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ ΔPQR, prove that* $\frac{AB}{PQ} = \frac{AD}{PM}$.

Sol. Proof: ΔABC ~ ΔPQR [Given]

$$\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$



$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} \quad ...(i) \quad \left(\because BD = \frac{1}{2}BC \text{ and } QM = \frac{1}{2}QR\right)$$

Also ∠B = ∠Q ...(*ii*) [∵ ΔABC ~ ΔPQR]

From (*i*) and (*ii*), we get

ΔABD ~ ΔPQM

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}.$$

Hence proved.

Exerise 6.4

1. *Let Δ ABC ~ ΔDEF and their areas be, respectively, 64 cm² and 121 cm². If EF = 15.4 cm, find BC.*

Sol. $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$

$\Rightarrow \quad \frac{8}{11} = \frac{BC}{15.4}$

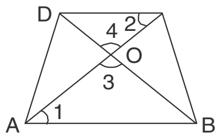
$\Rightarrow \quad BC = 11.2$ cm.

2. *Diagonals of a trapezium ABCD with AB ∥ DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.*

Sol. $\Delta AOB \sim \Delta COD$ ...(*i*)

As $\angle 1 = \angle 2$ [Alternate angles]

and $\angle 3 = \angle 4$ [Vertically opposite angles]



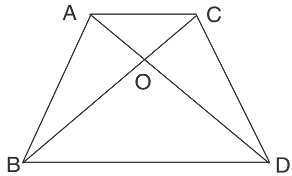
$\therefore \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$

[If triangles are similar, their areas are proportional to the squares of the corresponding sides]

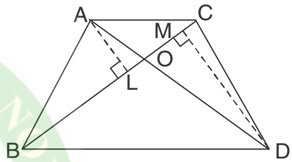
$= \frac{(2CD)^2}{CD^2} = \frac{4}{1}$

$\therefore \quad ar(\Delta AOB) : ar(\Delta COD) = 4 : 1.$

3. *In figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that* $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



Sol.



$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2}\,BC\cdot AL}{\frac{1}{2}\,BC\cdot DM}$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AL}{DM} \qquad ...(i)$$

But, $\Delta ALO \sim \Delta DMO$ [AA similarity]

$$\therefore \quad \frac{AL}{DM} = \frac{AO}{DO} \qquad ...(ii)$$

From (*i*) and (*ii*), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}.$$

Hence proved.

4. *If the areas of two similar triangles are equal, prove that they are congruent.*

Sol. Let $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \qquad ...(i)$$

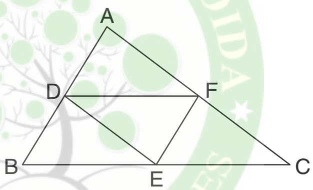
Also $ar(\Delta ABC) = ar(\Delta PQR)$ ...(*ii*) (given)

$$\therefore \quad \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \qquad \text{[From } (i), (ii)\text{]}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \Rightarrow \Delta ABC \cong \Delta PQR \quad \text{[SSS similarity]}$$

5. *D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find the ratio of the areas of Δ DEF and Δ ABC.*

Sol.



D and F are the mid-points of sides AB and AC of a ΔABC.

$$\therefore \quad DF \parallel BC \text{ and } DF = \frac{1}{2} BC$$

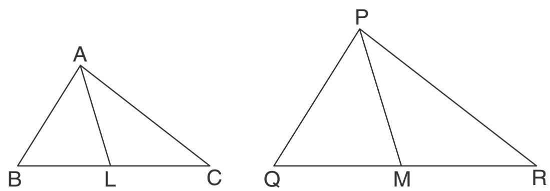
Also $\Delta EDF \sim \Delta ACB$. [AAA similarity]

$$\therefore \quad \frac{ar(\Delta DEF)}{ar(\Delta CAB)} = \frac{DF^2}{BC^2} = \frac{DF^2}{(2DF)^2} = \frac{1}{4}$$

$\therefore \quad ar(\Delta DEF) : ar(\Delta CAB) = 1 : 4.$

6. *Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.*

Sol.



$\Delta ABC \sim \Delta PQR$

$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} \qquad \Rightarrow \quad \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BL}{QM}$ ...(*i*) [$\because$ AL and PM are median]

Also $\angle B = \angle Q$ ...(*ii*) [$\because$ $\Delta ABC \sim \Delta PQR$]

From (*i*) and (*ii*),

$\Delta ABL \sim \Delta PQM$ [SAS similarity]

$\therefore \quad \frac{AB}{PQ} = \frac{AL}{PM}$

Hence $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AL}{PM}$ ...(*iii*)

Also $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ...(*iv*)

[If triangles are similar their areas are proportional to the squares of the corresponding sides.]

From (*iii*) and (*iv*), we get

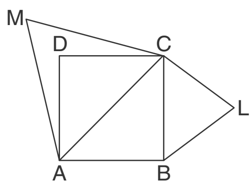
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AL^2}{PM^2}.$$

7. *Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.*

Sol. ΔBCL and ΔACM are equilateral triangles.

$\Delta BCL \sim \Delta ACM$ (AAA-Similarity)

$\Rightarrow \quad \frac{ar(\Delta BCL)}{ar(\Delta ACM)} = \frac{BC^2}{AC^2}$ ...(*i*)



Also $AC = \sqrt{2}\,BC$. ...(*ii*)

From (*i*) and (*ii*), we get

$$\frac{ar(\Delta BCL)}{ar(\Delta ACM)} = \frac{BC^2}{(\sqrt{2}BC)^2} = \frac{1}{2}$$

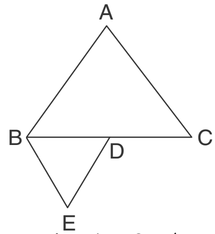
$\therefore \qquad ar(\Delta BCL) = \frac{1}{2}\, ar(\Delta ACM).$

8. *ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is*

(A) 2 : 1 (B) 1 : 2 (C) 4 : 1 (D) 1 : 4

Sol. As ΔABC and ΔBDE are equilateral.

$\therefore$ ΔABC ~ ΔBDE. Also BC = 2BD.



$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{BC^2}{BD^2} = \frac{(2BD)^2}{BD^2} = \frac{4}{1}.$$

$\therefore$ Option (C) is correct.

9. *Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio*

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

Sol. $\frac{\text{Area of first triangle}}{\text{Area of second triangle}} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}.$

$\therefore$ Option (D) is correct.

Exerise 6.5

1. *Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.*

(*i*) *7 cm, 24 cm, 25 cm*

(*ii*) *3 cm, 8 cm, 6 cm*

(*iii*) *50 cm, 80 cm, 100 cm*

(*iv*) *13 cm, 12 cm, 5 cm*

Sol. If square of one side is equal to the sum of the squares of the other two, then we say, "the sides form a right-angled triangle."

(*i*) $(25)^2 = (7)^2 + (24)^2 \Rightarrow 625 = 49 + 576.$

Hence, the sides are of a right-angled triangle.

The length of the hypotenuse is 25 cm.

(*ii*) $(8)^2 \neq (3)^2 + (6)^2 \Rightarrow 64 \neq 9 + 36,$

Hence, the sides are not of a right-angled triangle.

(*iii*) $(100)^2 \neq (50)^2 + (80)^2 \Rightarrow 10000 \neq 2500 + 6400$

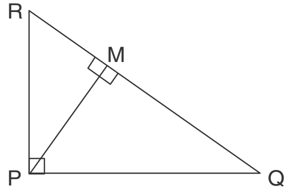
Hence, the sides are not of a right-angled triangle.

(*iv*) $(13)^2 = (12)^2 + (5)^2 \Rightarrow 169 = 144 + 25$

Hence, the sides are of a right-angled triangle.

The length of the hypotenuse is 13 cm.

2. *PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that $PM^2 = QM \cdot MR$.*



Sol. Let $\angle Q = x$

$\Rightarrow \quad \angle MPQ = 90° - x$

And $\angle RPM = 90° - (90° - x) = x$

$\Rightarrow \quad \angle Q = \angle RPM$

In ΔPMQ and ΔPMR,

$\angle Q = \angle RPM$ [Proved above]

$\angle PMQ = \angle PMR$

$\therefore \quad \Delta PQM \sim \Delta RPM$ [AA similarity]

$$\Rightarrow \quad \frac{PM}{RM} = \frac{QM}{PM} \quad \Rightarrow \quad PM^2 = QM \cdot MR.$$

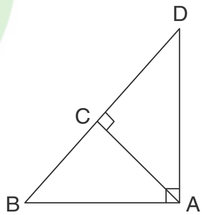
Hence proved.

3. *In figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that:*

(*i*) $AB^2 = BC \cdot BD$

(*ii*) $AC^2 = BC \cdot DC$

(*iii*) $AD^2 = BD \cdot CD$



Sol. (*i*) In ΔABC and ΔABD,

∠B is common.

$\angle ACB = \angle BAD$ [90° each]

$\Rightarrow \quad \Delta BAD \sim \Delta BCA$ [AA similarity]

$$\therefore \quad \frac{AB}{BD} = \frac{CB}{BA} \quad \Rightarrow \quad AB^2 = BC \cdot BD.$$

(*ii*) In ΔABC and ΔACD

$\angle ADC = \angle CAB$

$\angle ACD = \angle ACB$ [90° each]

$\Rightarrow \quad \Delta ABC \sim \Delta DAC$ [AA similarity]

$\Rightarrow \quad \frac{AC}{BC} = \frac{DC}{AC} \quad \Rightarrow \quad AC^2 = BC.DC.$

(*iii*) In ΔABD and ΔCAD,

$\angle BAD = \angle ACD$ [90° each]

and ∠D is common.

$\Rightarrow \quad \Delta ABD \sim \Delta CAD$ [AA similarity]

$\therefore \quad \frac{AD}{CD} = \frac{BD}{AD}$

$\Rightarrow \quad AD^2 = BD \,.\, CD.$ **Hence proved.**

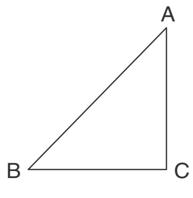
4. *ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.*

Sol. ΔABC is right angled at C.

$\Rightarrow \quad AB^2 = AC^2 + BC^2.$ Also AC = BC.

$\therefore \quad AB^2 = AC^2 + AC^2 = 2AC^2.$ **Hence proved.**

5. *ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.*



Sol. $AC = BC$ ...(*i*)

$AB^2 = 2AC^2 = AC^2 + AC^2$

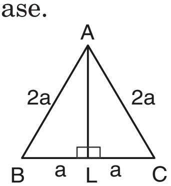
$= AC^2 + BC^2$ [From (*i*)]

$\Rightarrow$ ΔABC is right-angled at C.

[Converse of Pythagoras theorem]

6. *ABC is an equilateral triangle of side 2a. Find each of its altitudes.*

Sol. In equilateral triangle, altitude bisects the base.



$\therefore \quad AL = \sqrt{(2a)^2 - (a)^2} = \sqrt{4a^2 - a^2} = \sqrt{3}\,a.$

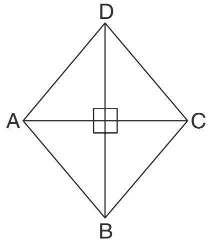
Similarly, we can find out that each of other two altitudes as $\sqrt{3}\,a$.

7. *Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.*

Sol. ABCD is a rhombus and we know diagonals of a rhombus bisect each other at right angles.

ΔOAD is right-angled at O.

$\therefore \quad AD^2 = OA^2 + OD^2 \quad ...(i)$



Similarly,

$AB^2 = OA^2 + OB^2 \quad ...(ii)$

$BC^2 = OB^2 + OC^2 \quad ...(iii)$

$CD^2 = OC^2 + OD^2 \quad ...(iv)$

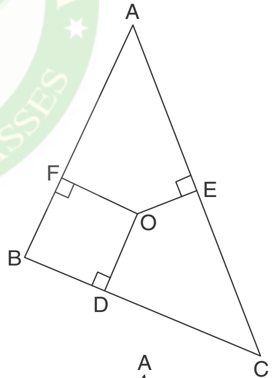
From (*i*), (*ii*), (*iii*) and (*iv*), we get

$$AB^2 + BC^2 + CD^2 + DA^2 = 2[OA^2 + OB^2 + OC^2 + OD^2]$$

$$= 2\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2 + \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right]$$

$$= AC^2 + BD^2.$$

8. *In figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that*



(*i*) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2,$

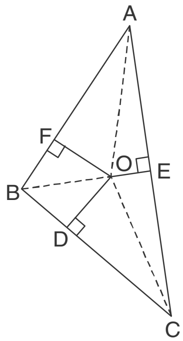
(*ii*) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2.$

Sol. (*i*) In right triangles AOF, BOD and COE, we have respectively

$OA^2 = AF^2 + OF^2$

$OB^2 = BD^2 + OD^2$

and $OC^2 = CE^2 + OE^2.$



$\Rightarrow \quad AF^2 + BD^2 + CE^2$

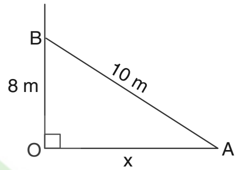
$= OA^2 + OB^2 + OC^2 - OF^2 - OD^2 - OE^2$

(*ii*) $\quad OB^2 - OC^2 = BD^2 - CD^2$

$$OC^2 - OA^2 = CE^2 - AE^2$$
$$OA^2 - OB^2 = AF^2 - BF^2.$$
$$\Rightarrow \quad 0 = BD^2 - CD^2 + CE^2 - AE^2 + AF^2 - BF^2$$
$$\Rightarrow \quad BD^2 + CE^2 + AF^2 = CD^2 + AE^2 + BF^2.$$

9. *A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.*

Sol. Window is at B.



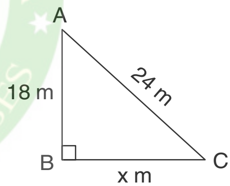
AB = 10 m, OB = 8 m

Distance $x = \sqrt{(10)^2 - (8)^2}$ m

$= \sqrt{36}$ m = 6 m.

10. *A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?*

Sol. AB is a pole of 18 m.



AC is a guy wire of 24 m.

BC = x m

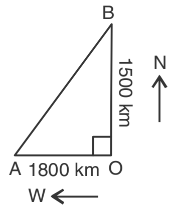
In ΔABC,

$$AC^2 = (AB)^2 + (BC)^2$$
$$(24)^2 = (18)^2 + x^2$$
$$\Rightarrow \quad x^2 = 576 - 324 \quad \Rightarrow \quad x^2 = 252$$
$$\Rightarrow \quad x = \sqrt{252}\text{ m} = 6\sqrt{7}\text{ m}.$$

11. *An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?*

Sol. Distance OB travelled due north from the starting point O in $1\frac{1}{2}$ hours is 1500 km.

Distance OA travelled due west from the starting point



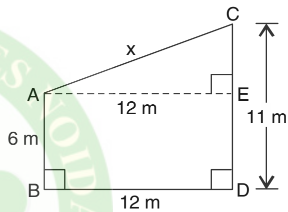
O in $1\frac{1}{2}$ hours is 1800 km.

$\therefore \quad AB = \sqrt{(1500)^2 + (1800)^2}$ km

$= 100\sqrt{225 + 324}$ km

$= 100\sqrt{549}$ km $= 300\sqrt{61}$ km.

12. *Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.*



Sol. Draw AE || BD, meeting CD at E.

$\therefore \quad CE = (11 - 6)$ m $= 5$ m

and AE = BD = 12 m.

In right-angled triangle AEC,

$AC^2 = AE^2 + CE^2$

$\Rightarrow \quad x = \sqrt{(12)^2 + (5)^2}$ m = 13 m.

13. *D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.*

Sol. Using Pythagoras theorem in right triangles ΔAEC, ΔBDC, ΔABC and ΔDEC, we have respectively.

$AE^2 = EC^2 + AC^2$

$BD^2 = BC^2 + CD^2$

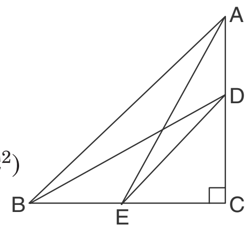
$AB^2 = BC^2 + AC^2$

and $DE^2 = CD^2 + AC^2$

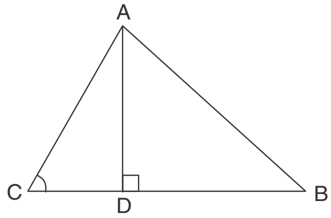
$AE^2 + BD^2 = EC^2 + AC^2 + BC^2 + CD^2$

$= (EC^2 + DC)^2 + (AC^2 + BC^2)$

$= DE^2 + AB^2$.



14. *The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD (see figure). Prove that $2AB^2 = 2AC^2 + BC^2$.*



Sol. $BD = 3\ CD \quad \Rightarrow BD = \frac{3}{4}BC,\ CD = \frac{1}{4}BC$

$$AB^2 - AC^2 = (BD^2 + AD^2) - (AD^2 + CD^2)$$

$$= BD^2 - CD^2 = \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{1}{2}BC^2$$

$\Rightarrow \quad 2AB^2 - 2AC^2 = BC^2.$

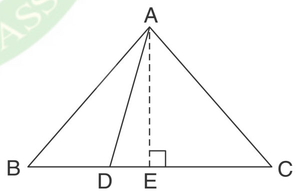
$\therefore \quad 2AB^2 = 2AC^2 + BC^2.$

15. *In an equilateral triangle ABC, D is a point on side BC such that* $BD = \frac{1}{3}BC$. *Prove that* $9AD^2 = 7AB^2$.

Sol. Given: In ΔABC, $AB = BC = CA$

D is a point on BC such that

$$BD = \frac{1}{3}BC$$



To prove: $9AD^2 = 7AB^2$

Construction: Draw $AE \perp BC$, which intersects BC at E.

Proof: As ΔABC is an equilateral triangle.

$$\therefore \quad BE = EC = \frac{BC}{2} = \frac{AB}{2} \quad ...(i)$$

Further, $DE = BE - BD = \frac{AB}{2} - \frac{BC}{3}$

[From construction]

$$= \frac{AB}{2} - \frac{AB}{3} = \frac{AB}{6} \quad ...(ii)$$

In right-angled triangle ABE,

$$AE^2 = AB^2 - BE^2 = AB^2 - \left(\frac{AB}{2}\right)^2 = \frac{3}{4}.AB^2 \quad ...(iii)$$

[From (i)]

Also, in right-angled triangle ADE,

$$AE^2 = AD^2 - DE^2 = AD^2 - \frac{AB^2}{36} \quad ...(iv)$$

[From (ii)]

From equations (iii) and (iv), we have

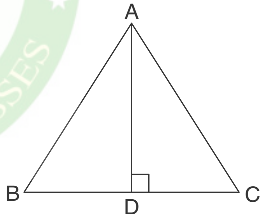
$$\frac{3}{4}AB^2 = AD^2 - \frac{AB^2}{36}$$

$\Rightarrow$ $7AB^2 = 9AD^2$. **Hence proved.**

16. *In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.*

Sol. Let side of an equilateral triangle be a and AD is altitude.

In right angled triangle ADB.



$$a^2 = AD^2 + \left(\frac{a}{2}\right)^2$$

$\Rightarrow$ $AD^2 = \frac{3}{4}a^2$

$\Rightarrow$ $4AD^2 = 3(AB)^2$.

17. *Tick the correct answer and justify: In*

Δ ABC, AB = $6\sqrt{3}$ cm,

AC = 12 cm and BC = 6 cm. The angle B is:

(A) *120°* (B) *60°*

(C) *90°* (D) *45°*

Sol. $AB^2 = 108$, $AC^2 = 144$, $BC^2 = 36$.

As $AC^2 = AB^2 + BC^2$.

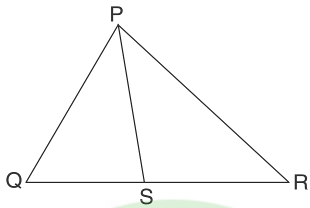
$\Rightarrow$ Triangle is right angled at B. Therefore, $\angle B = 90°$.

Hence, option (C) is correct.

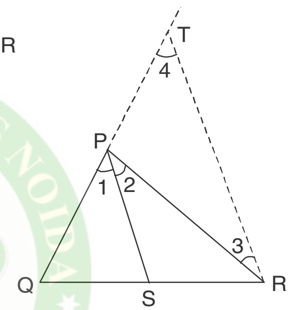
Exerise 6.6 (OPTIONAL)

1. *In figure, PS is the bisector of ∠QPR of ΔPQR. Prove that*

$$\frac{QS}{SR} = \frac{PQ}{PR}.$$



Sol. Given: In ΔPQR, PS is bisector of ∠QPR, meeting QR at S.



To prove: $\frac{QS}{SR} = \frac{PQ}{PR}$.

Construction: Draw RT || PS, meeting QP produced at T.

Proof: ∠1 = ∠4 ...(*i*) [Corresponding angles]

∠2 = ∠3 ...(*ii*) [Alternate angles]

Also ∠1 = ∠2 ...(*iii*) [∵ PS is bisector of ∠QPR]

⇒ ∠3 = ∠4 [From (*i*), (*ii*), (*iii*)]

⇒ PR = PT ...(*iv*)

In ΔQTR, PS || RT [Construction]

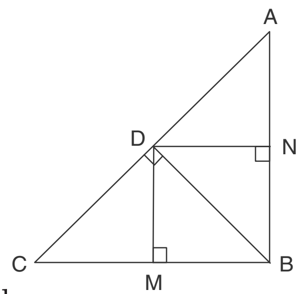
⇒ $\frac{QS}{SR} = \frac{QP}{PT}$ ⇒ $\frac{QS}{SR} = \frac{QP}{PR}$ [From (*i*)]

Hence proved.

2. *In figure, D is a point on hypotenuse AC of ΔABC, such that BD ⊥ AC, DM ⊥ BC and DN ⊥ AB. Prove that:*

(*i*) $DM^2 = DN \,.\, MC$

(*ii*) $DN^2 = DM \,.\, AN$



Sol. In quadrilateral DMBN

∠DMB = ∠MBN = ∠BND = 90° each

$\therefore$ $\angle MDN = 90°$

$\Rightarrow$ DMBN is a rectangle

$\Rightarrow$ MB = DN and DM = NB ...(*i*)

In triangle CMD and DMB,

$\angle CMD = \angle DMB = 90°$ each

$\angle C = \angle MDB$

$\therefore$ $\Delta CMD \sim \Delta DMB$

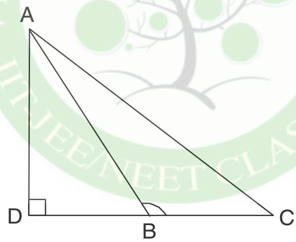
$\Rightarrow \frac{DM}{MC} = \frac{BM}{MD} \Rightarrow DM^2 = BM.MC = DN.MC$ [From (*i*)]

Similarly we can show

$\Delta AND \sim \Delta DNB$

and $DN^2 = DM.AN$.

3. *In figure, ABC is a triangle in which $\angle ABC > 90°$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC . BD$.*



Sol. In right-angled ΔADC, $AC^2 = AD^2 + CD^2$

[Pythagoras Theorem]

$\Rightarrow \quad AD^2 = AC^2 - CD^2$ ...(*i*)

Also, in right-angled ΔADB, $AB^2 = AD^2 + BD^2$

[Pythagoras Theorem]

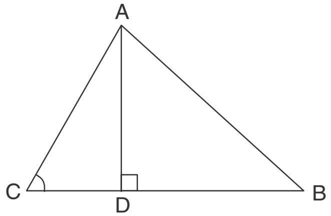
$\Rightarrow \quad AD^2 = AB^2 - BD^2$ ...(*ii*)

From (*i*) and (*ii*), we get

$$AC^2 = AB^2 + CD^2 - BD^2$$
$$= AB^2 + (BC + BD)^2 - BD^2$$
$$= AB^2 + BC^2 + 2BC . BD + BD^2 - BD^2$$
$$= AB^2 + BC^2 + 2BC.BD.$$

Hence proved.

4. *In figure, ABC is a triangle in which $\angle ABC < 90°$ and $AD \perp BC$. Prove that $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$.*



Sol. Consider right-angled triangle ADB.

$AB^2 = AD^2 + BD^2$ ...(*i*) [Pythagoras Theorem]

and in right-angled triangle ADC,

$AC^2 = AD^2 + DC^2$ ...(*ii*) [Pythagoras Theorem]

$\Rightarrow AC^2 - AB^2 = DC^2 - BD^2$ [From (*i*) and (*ii*)]

$\Rightarrow AC^2 = AB^2 + (BC - BD)^2 - BD^2$

$\Rightarrow AC^2 = AB^2 + BC^2 + BD^2 - 2BC.BD - BD^2$

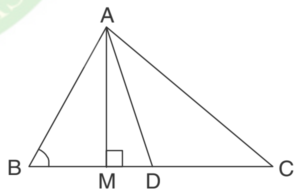
$\Rightarrow AC^2 = AB^2 + BC^2 - 2BC.BD.$

5. *In figure, AD is a median of a triangle ABC and $AM \perp BC$. Prove that:*

(*i*) $AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(*ii*) $AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$

(*iii*) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$



Sol. Consider right-angled triangle AMC.

(*i*) $AC^2 = AM^2 + MC^2$ ...(*i*) [Pythagoras Theorem]

Also in right-angled-triangle AMD,

$AD^2 = AM^2 + MD^2$ ...(*ii*) [Pythagoras Theorem]

$\Rightarrow AC^2 - AD^2 = MC^2 - MD^2$ [From (*i*), (*ii*)]

$= (MC + MD)(MC - MD)$

$= (DC + MD + MD)(DC)$

$= DC^2 + 2MD.DC$

$$= \left(\frac{BC}{2}\right)^2 + 2MD.\frac{BC}{2} \quad [\because \text{ D is mid-point of BC}]$$

$$\Rightarrow \quad AC^2 = AD^2 + MD.BC + \left(\frac{BC}{2}\right)^2.$$

(*ii*) Consider right-angled triangle AMD.

$$AB^2 = AM^2 + BM^2 \quad ...(i) \quad \text{[Pythagoras Theorem]}$$

Also in right-angled triangle AMD,

$$AD^2 = AM^2 + MD^2 \quad ...(ii) \quad \text{[Pythagoras Theorem]}$$

$$\Rightarrow \quad AB^2 - AD^2 = BM^2 - MD^2 \quad \text{[From (i) and (ii)]}$$

$$= (BM + MD)(BM - MD)$$

$$= (BD)(BD - MD - MD)$$

$$\Rightarrow \quad AB^2 = AD^2 + BD^2 - 2BD.MD$$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2 . \frac{BC}{2} . MD$$

$$[\because \text{ D is mid-point of BC}]$$

$$\Rightarrow \quad AB^2 = AD^2 + \left(\frac{BC}{2}\right)^2 - BC.DM.$$

(*iii*) Adding result (*i*) and (*ii*), we get

$$AC^2 + AB^2 = 2AD^2 + \frac{1}{2}(BC)^2.$$

6. *Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.*

Sol. Draw CL ⊥ AB and DM ⊥ AB.

In triangles AMD and BLC,

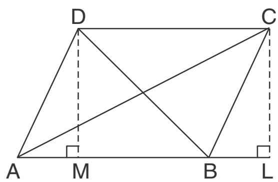
AD = BC [Opposite sides of a $||^{gm}$]

DM = CL [Distance between two parallel lines]

∠DMA = ∠CLB = [90° each]

∴ ΔAMD ≅ ΔBLC [RHS similarity]

⇒ AM = BL ...(*i*)



In right-angled triangle ALC,

$$AC^2 = AL^2 + CL^2 \quad (ii)$$

In right-angled triangle BLC,

$$BC^2 = BL^2 + CL^2 \quad ...(iii)$$

$$\Rightarrow \quad AC^2 - BC^2 = AL^2 - BL^2 \quad \text{[From } (ii) \text{ and } (iii)]$$

$$= (AL + BL)(AL - BL)$$

$$= (AB + BL + BL)(AB)$$

$$= AB^2 + 2BL.AB$$

$$\Rightarrow \quad AC^2 = BC^2 + AB^2 + 2BL.AB \quad ...(iv)$$

In right-angled triangle DMB,

$$DB^2 = DM^2 + MB^2 \quad ...(v)$$

and in right-angled triangle DMA,

$$AD^2 = DM^2 + AM^2 \quad ...(vi)$$

$$\Rightarrow \quad DB^2 - AD^2 = MB^2 - AM^2 \quad ...\text{[From } (v) \text{ and } (vi)]$$

$$= (MB - AM)(MB + AM)$$

$$= (AB - AM - AM)(AB)$$

$$= AB^2 - 2AM.AB$$

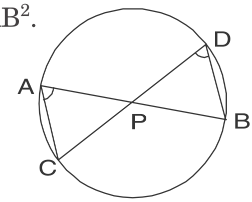
$$\Rightarrow \quad BD^2 = AD^2 + AB^2 - 2AM.AB \quad ...(vii)$$

$$\therefore \quad AC^2 + BD^2 = BC^2 + AB^2 + AD^2 + AB^2$$

[From (*iv*), (*vii*) and (*i*)]

$$\Rightarrow \quad AC^2 + BD^2 = BC^2 + DC^2 + AD^2 + AB^2.$$

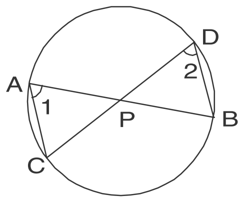
7. *In figure, two chords AB and CD intersect each other at the point P. Prove that:*



(*i*) $\Delta APC \sim \Delta DPB$

(*ii*) $AP \cdot PB = CP \cdot DP$

Sol. Given: Chords AB and CD of given circle intersect at P.



To prove: $\Delta APC \sim \Delta DPB$.

Proof: $\angle 1 = \angle 2$

[Angles in the same segment]

$\angle APC = \angle DPB$

[Vertically opposite angles]

(*i*) $\therefore \quad \Delta APC \sim \Delta DPB$ [AA similarity]

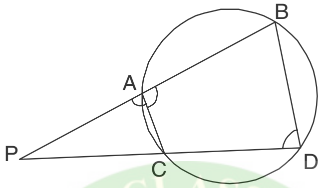
(*ii*) $$\frac{AP}{PC} = \frac{DP}{PB}$$ [Corresponding sides]

$\Rightarrow$ AP . PB = CP . DP. **Hence proved.**

8. *In figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that:*

(*i*) *Δ PAC ~ Δ PDB*

(*ii*) *PA . PB = PC . PD*



Sol. Given: Chords AB and CD intersect at P outside the circle.

Proof: $\angle 1 + \angle 2 = 180°$ ...(*a*) [Linear pair]

$\angle 2 + \angle 3 = 180°$ ...(*b*)

[Sum of pair of opposite angles of a cyclic quadrilateral]

From (*a*) and (*b*), we get

$\angle 1 + \angle 2 = \angle 2 + \angle 3 \Rightarrow \angle 1 = \angle 3$ ...(*c*)

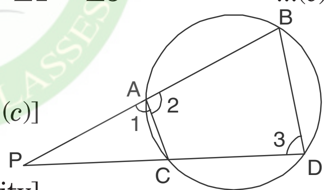
(*i*) In ΔPAC and ΔPBD,

$\angle$P is common.

$\angle 1 = \angle 3$ [From (*c*)]

$\therefore$ $\Delta PAC \sim \Delta PDB$

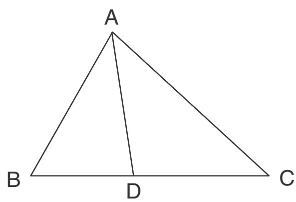
[AA similarity]



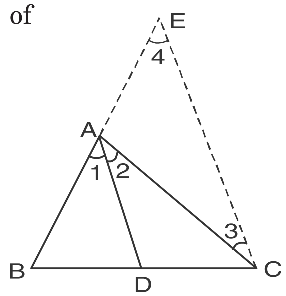
(*ii*) $\dfrac{AP}{CP} = \dfrac{DP}{BP}$ [Corresponding sides]

$\Rightarrow$ AP . PB = CP . DP. **Hence proved.**

9. *In figure, D is a point on side BC of Δ ABC such that $\dfrac{BD}{CD} = \dfrac{AB}{AC}$. Prove that AD is the bisector of $\angle$BAC.*



Sol. **Given:** D is a point on the side BC of a ΔABC such that $\frac{BD}{DC} = \frac{AB}{AC}$.



To prove: AD is bisector of $\angle BAC$

Construction: Draw CE || DA, meeting BA produced at E.

Proof: In ΔBEC, AD || CE

$\Rightarrow \quad \frac{BD}{DC} = \frac{AB}{AE}$ ...(*i*) [Basic Proportionality Theorem]

Also $\quad \frac{BD}{DC} = \frac{AB}{AC}$ ...(*ii*) [Given]

$\Rightarrow \quad \frac{AB}{AE} = \frac{AB}{AC}$ [From (*i*) and (*ii*)]

$\Rightarrow \quad AE = AC$

In ΔACE, as AE = AC [Proved above]

$\Rightarrow \quad \angle 3 = \angle 4$ ...(*iii*)

Also $\quad \angle 3 = \angle 2$ ...(*iv*) [Alternate angles]

and $\quad \angle 1 = \angle 4$ ...(*v*) [Corresponding angles]

From (*iii*), (*iv*) and (*v*), we get

$\angle 1 = \angle 2$

$\Rightarrow$ AD is the bisector of $\angle BAC$.

10. *Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?*

Sol. $BO = \sqrt{(2.4)^2 + (1.8)^2}$ m

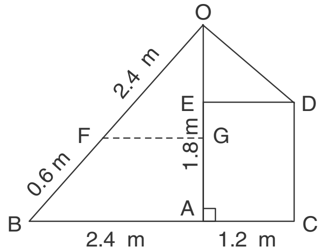
$= \sqrt{5.76 + 3.24}$ m $= \sqrt{9.00} = 3$ m

After 12 second, let string is at F,

Distance BF covered in 12 seconds

$= 5 \times 12 \text{ cm} = 60 \text{ cm} = 0.6 \text{ m}$

$\therefore$ OF = BO – BF = 3 – 0.6 = 2.4 m



As $\Delta OBA \sim \Delta OFG$,

We have $\frac{OB}{OF} = \frac{AB}{FG} \Rightarrow \frac{3}{2.4} = \frac{2.4}{FG}$

$\Rightarrow \quad FG = \frac{5.76}{3} = 1.92 \text{ m}$

$\therefore$ Horizontal distance of the fly

$= FG + AC$

$= (1.92 + 1.2) \text{ m} = 3.12 \text{ m}.$