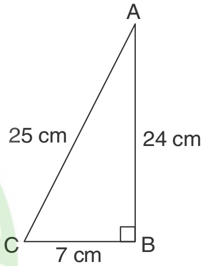
Exerise 8.1

1. *In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:*

(i) sin A, cos A

(ii) sin C, cos C

Sol. AB = 24 cm, BC = 7 cm, using Pythagoras Theorem,



$$AC = \sqrt{24^2 + 7^2} = \sqrt{576 + 49}$$

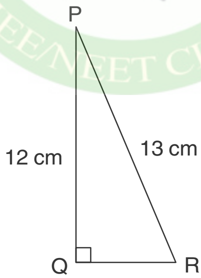
$$= \sqrt{625} = 25 \text{ cm.}$$

(i) $\sin A = \frac{BC}{AC} = \frac{7}{25}$,

$\cos A = \frac{AB}{AC} = \frac{24}{25}$.

(ii) $\sin C = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{BC}{AC} = \frac{7}{25}$.

2. *In figure, find tan P – cot R.*



Sol. PQ = 12 cm, PR = 13 cm.
Using Pythagoras Theorem,

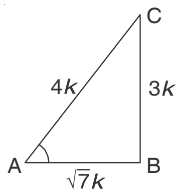
$$RQ = \sqrt{PR^2 - PQ^2} = \sqrt{13^2 - 12^2}$$

$$= \sqrt{169 - 144} = \sqrt{25} = 5 \text{ cm}$$

$\tan P = \frac{RQ}{PQ} = \frac{5}{12}$, $\cot R = \frac{RQ}{PQ} = \frac{5}{12}$.

$\therefore \tan P = \cot R \Rightarrow \tan P - \cot R = 0.$

3. *If sin A = $\frac{3}{4}$, calculate cos A and tan A.*



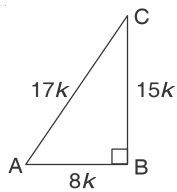
Sol. $\sin A = \frac{3}{4} = \frac{3k}{4k}$

$$AB = \sqrt{16k^2 - 9k^2} = \sqrt{7}\,k$$

$$\therefore \quad \cos A = \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}.$$

4. *Given 15 cot A = 8, find sin A and sec A.*



Sol. $15 \cot A = 8 \Rightarrow \cot A = \frac{8}{15}$

Let $AB = 8k$, $BC = 15k$, then

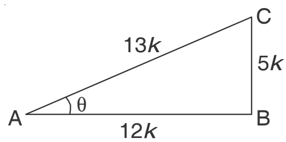
$$AC = \sqrt{AB^2 + BC^2} = \sqrt{64k^2 + 225k^2}$$

$$= \sqrt{289k^2} = 17k$$

$$\therefore \quad \sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17},$$

$$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}.$$

5. *Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.*



Sol. $\sec \theta = \frac{13}{12} = \frac{13k}{12k}$, *i.e.,* $\frac{AC}{AB}$

$$\therefore \quad BC = \sqrt{169k^2 - 144k^2} = 5k$$

$$\sin \theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}, \quad \cos \theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12},\ \cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

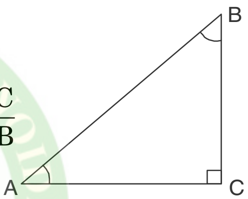
$$\text{and } \operatorname{cosec}\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}.$$

6. *If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.*

Sol. ∠A and ∠B are two acute angles either in same right-angled triangle or in different.

Case I: Let ∠A and ∠B are acute angles in right ΔABC.

$$\cos A = \frac{AC}{AB} \text{ and } \cos B = \frac{BC}{AB}$$



As given cos A = cos B $\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$

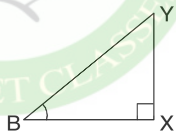
$\Rightarrow$ AC = BC

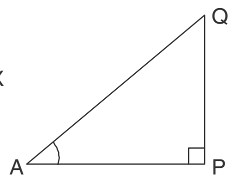
$\therefore$ ∠B = ∠A. [Angles opposite to equal sides are equal]

Case II: Let ∠A belong to rt ΔAPQ and ∠B belong to rt ΔBXY.

In ΔAPQ,

$$\cos A = \frac{AP}{AQ} \qquad ...(i)$$





In ΔBXY,

$$\cos B = \frac{BX}{BY} \qquad ...(ii)$$

As given cos A = cos B $\Rightarrow \frac{AP}{AQ} = \frac{BX}{BY}$

[From (*i*) and (*ii*)]

$$\Rightarrow \frac{AP}{BX} = \frac{AQ}{BY} = k \text{ (say)}$$

$\therefore$ AP = k BX and AQ = k BY

$$\text{Now, } \frac{PQ}{XY} = \frac{\sqrt{AQ^2 - AP^2}}{\sqrt{BY^2 - BX^2}} = \frac{\sqrt{k^2BY^2 - k^2BX^2}}{\sqrt{BY^2 - Bk^2}}$$

$$\therefore \qquad \frac{PQ}{XY} = \frac{k\sqrt{BY^2 - BX^2}}{\sqrt{BY^2 - Bk^2}} = k$$

Therefore, $\dfrac{AP}{BX} = \dfrac{AQ}{BY} = \dfrac{PQ}{XY}$

$\therefore \qquad \Delta APQ \sim \Delta BXY$ [By SSS similarity theorem]

$\therefore \qquad \angle A = \angle B.$

7. *If* $\cot\ \theta = \dfrac{7}{8}$, *evaluate*:

(*i*) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$, (*ii*) $\cot^2\ \theta$

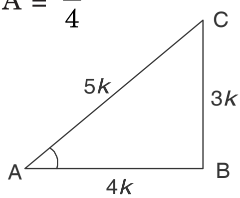
Sol. (*i*) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{1-\sin^2\theta}{1-\cos^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta}$

$$= \cot^2\ \theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}.$$

(*ii*) $\cot^2\ \theta = \left(\dfrac{7}{8}\right)^2 = \dfrac{49}{64}.$

8. *If* $3\cot A = 4$, *check whether* $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ *or not.*

Sol. $3\cot A = 4 \Rightarrow \cot A = \dfrac{4}{3}$ or $\tan A = \dfrac{3}{4}$



If AB = $4k$ and BC = $3k$, then

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(4k)^2 + (3k)^2}$$

$$= \sqrt{25k^2} = 5k$$

$\therefore \quad \cos A = \dfrac{AB}{AC} = \dfrac{4}{5}$ and $\sin A = \dfrac{BC}{AC} = \dfrac{3}{5}$

$$\text{LHS} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{16-9}{16+9} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16-9}{25} = \frac{7}{25}$$

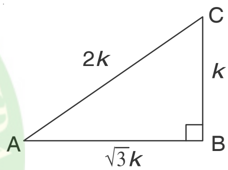
Hence, $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A.$

9. *In triangle ABC, right-angled at B, if* $\tan A = \dfrac{1}{\sqrt{3}}$, *find the value of*:

(*i*) *sin A cos C + cos A sin C*

(*ii*) *cos A cos C – sin A sin C*

Sol. $\tan A = \dfrac{1}{\sqrt{3}} \Rightarrow AB = \sqrt{3}k$ and $BC = k$, then



$$AC = \sqrt{AB^2 + BC^2} = \sqrt{(\sqrt{3}k)^2 + k^2}$$

$$= \sqrt{3k^2 + k^2} = 2k.$$

$$\sin A = \frac{BC}{AC} = \cos C \text{ and } \sin C = \frac{AB}{AC} = \cos A$$

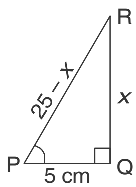
(*i*) $\sin A \cos C + \cos A \sin C = \dfrac{1}{2} \cdot \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2}$

$$= \frac{1}{4} + \frac{3}{4} = 1.$$

(*ii*) $\cos A \cos C - \sin A \sin C = \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{3}}{2}$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0.$$

10. *In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.*



Sol. PQ = 5 cm and PR + QR = 25 cm

Let RQ = x, then PR = 25 – x

Using Pythagoras theorem,

$$PR^2 = PQ^2 + RQ^2$$

$$(25 - x)^2 = x^2 + (5)^2 \Rightarrow 625 - 50x + x^2 = x^2 + 25$$

$$\Rightarrow \quad 50x = 600 \Rightarrow x = 12$$

$\therefore$ RQ = 12 cm and PR = 13 cm.

$$\therefore \quad \sin P = \frac{12}{13}, \cos P = \frac{5}{13} \text{ and } \tan P = \frac{12}{5}.$$

11. *State whether the following are true or false. Justify your answer.*

(*i*) *The value of tan A is always less than 1.*

(*ii*) *sec A =* $\frac{12}{5}$ *for some value of angle A.*

(*iii*) *cos A is the abbreviation used for the cosecant of angle A.*

(*iv*) *cot A is the product of cot and A.*

(*v*) *sin* $\theta = \frac{4}{3}$ *for some angle* θ.

Sol. (*i*) False, As $\tan A = \frac{p}{b}$ where perpendicular is not always less than base in a right-angled triangle.

(*ii*) True, $\sec A = \frac{12}{5}$, true as $\cos A = \frac{5}{12} < 1$, true.

(*iii*) False, cos A is abbreviation used for the cosine of angle A.

(*iv*) False, cot A $\neq$ cot $\times$ A

(*v*) False, as sin θ is always less than or equal to 1.

Exerise 8.2

1. *Evaluate the following:*

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2 \tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \text{cosec } 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \text{cosec } 60°}{\sec 30° + \cos 60° + \cot 45°}$ (v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Sol. (i) $\dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \cdot \dfrac{1}{2} = \dfrac{3}{4} + \dfrac{1}{4} = 1$

(ii) $2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2 = 2 + \dfrac{3}{4} - \dfrac{3}{4} = 2$

(iii) $$\dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2 + 2\sqrt{3}}$$

$$= \dfrac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)} \times \dfrac{(\sqrt{3}-1)\sqrt{2}}{(\sqrt{3}-1)\sqrt{2}}$$

$$= \dfrac{(3-\sqrt{3})\sqrt{2}}{2 \times 2 \times (3-1)} = \dfrac{3\sqrt{2} - \sqrt{6}}{8}.$$

(iv) $$\dfrac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \dfrac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{3}{2}} = \dfrac{3\sqrt{3} - 4}{3\sqrt{3} + 4} = \dfrac{(3\sqrt{3} - 4)^2}{27 - 16}$$

[Rationalising the denominator]

$$= \dfrac{27 + 16 - 24\sqrt{3}}{11} = \dfrac{43 - 24\sqrt{3}}{11}.$$

(v) $$\dfrac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \dfrac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \dfrac{15 + 64 - 12}{12} \times \dfrac{4}{1 + 3} = \dfrac{67}{12}.$$

2. *Choose the correct option and justify your choice:*

(*i*) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$ (B) $\cos 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

(*ii*) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A) $\tan 90^\circ$ (B) 1 (C) $\sin 45^\circ$ (D) 0

(*iii*) *sin 2A = 2 sin A is true when A =*

(A) 0° (B) 30° (C) 45° (D) 60°

(*iv*) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ} =$

(A) $\cos 60^\circ$ (B) $\sin 60^\circ$ (C) $\tan 60^\circ$ (D) $\sin 30^\circ$

Sol. (*i*) (A). $\dfrac{2\cdot\dfrac{1}{\sqrt{3}}}{1+\left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{\dfrac{2}{\sqrt{3}}}{1+\dfrac{1}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{4} = \dfrac{\sqrt{3}}{2} = \sin 60^\circ.$

(*ii*) (D). $\dfrac{1-(1)^2}{1+(1)^2} = \dfrac{0}{2} = 0.$

(*iii*) (A). Because sin 2A = sin 0 = 0
and 2 sin A = 2 sin 0° = 0.

(*iv*) (C). $\dfrac{2\times\dfrac{1}{\sqrt{3}}}{1-\left(\dfrac{1}{\sqrt{3}}\right)^2} = \dfrac{\dfrac{2}{\sqrt{3}}}{1-\dfrac{1}{3}} = \dfrac{2}{\sqrt{3}} \times \dfrac{3}{2} = \sqrt{3} = \tan 60^\circ.$

3. *If* $\tan(A+B) = \sqrt{3}$ *and* $\tan(A-B) = \dfrac{1}{\sqrt{3}}$; $0^\circ < A+B \le 90^\circ$; $A > B$, *find A and B.*

Sol. $\tan(A+B) = \sqrt{3} = \tan 60^\circ \Rightarrow A+B = 60^\circ$ ...(*i*)

$\tan(A-B) = \dfrac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow A-B = 30^\circ$...(*ii*)

Solving (*i*) and (*ii*), we get A = 45° and B = 15°.

4. *State whether the following are true or false. Justify your answer.*

(*i*) *sin (A + B) = sin A + sin B.*

(*ii*) *The value of sin θ increases as θ increases.*

(*iii*) *The value of cos θ increases as θ increases.*

(*iv*) *sin θ = cos θ for all values of θ.*

(*v*) *cot A is not defined for A = 0°.*

Sol. (*i*) False. Let A = 30°, B = 60°

$\sin (A + B) = \sin (30° + 60°) = \sin 90° = 1$

$$\sin A + \sin B = \sin 30° + \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

$\therefore \quad \sin (A + B) \neq \sin A + \sin B.$

(*ii*) True. As $\sin 0° = 0$, $\sin 30° = \frac{1}{2}$ and $\sin 90° = 1$.

(*iii*) False. As $\cos 0° = 1$, $\cos 60° = \frac{1}{2}$ and $\cos 90° = 0$.

(*iv*) False. Only for θ = 45°, cos θ = sin θ.

(*v*) True. $\cot A = \frac{\cos A}{\sin A}$

$\Rightarrow \quad \cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ which is not defined.

Exerise 8.3

1. *Evaluate*:

(*i*) $\frac{\sin 18°}{\cos 72°}$ (*ii*) $\frac{\tan 26°}{\cot 64°}$

(*iii*) *cos 48° – sin 42°* (*iv*) *cosec 31° – sec 59°*

Sol. (*i*) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°} = \frac{\cos 72°}{\cos 72°} = 1.$

[$\because$ sin (90° – θ) = cos θ]

(ii) $\dfrac{\tan 26°}{\cot 64°} = \dfrac{\tan 26°}{\cot (90° - 26°)} = \dfrac{\tan 26°}{\tan 26°} = 1$

[∵ $\cot (90° - \theta) = \tan \theta$]

(iii) $\cos 48° - \sin 42° = \cos (90° - 42°) - \sin 42°$

$= \sin 42° - \sin 42° = 0.$

[∵ $\cos (90° - \theta) = \sin \theta$]

(iv) $\text{cosec } 31° - \sec 59° = \text{cosec } 31° - \sec (90° - 31°)$

$= \text{cosec } 31° - \text{cosec } 31° = 0.$

[∵ $\sec (90° - \theta) = \text{cosec } \theta$]

2. *Show that:*

(i) *$\tan 48° \tan 23° \tan 42° \tan 67° = 1$*

(ii) *$\cos 38° \cos 52° - \sin 38° \sin 52° = 0.$*

Sol. (i) LHS $= \tan 48° \tan 23° \tan 42° \tan 67°$

$= \tan 48° \tan 23° \tan (90° - 48°) \tan (90° - 23°)$

$= \tan 48° \tan 23° \cot 48° \cot 23°$

$= \tan 48° \tan 23° \times \dfrac{1}{\tan 48°} \times \dfrac{1}{\tan 23°}$

$= 1 =$ RHS.

(ii) LHS $= \cos 38° \cos 52° - \sin 38° \sin 52°$

$= \cos (90° - 52°) \cos (90° - 38°) - \sin 38° \sin 52°$

$= \sin 52° \sin 38° - \sin 38° \sin 52° = 0 =$ RHS.

3. *If $\tan 2A = \cot(A - 18°)$, where 2A is an acute angle, find the value of A.*

Sol. $\tan 2A = \cot (A - 18°) = \tan \{90° - (A - 18°)\}$

$\Rightarrow 2A = 90° - A + 18° \Rightarrow 3A = 108°$

$\Rightarrow A = \dfrac{108°}{3} = 36°.$

4. *If $\tan A = \cot B$, prove that $A + B = 90°$.*

Sol. $\tan A = \cot B = \tan (90° - B)$

$\Rightarrow A = 90° - B \Rightarrow A + B = 90°$. **Hence proved.**

5. *If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.*

Sol. sec 4A = cosec (A – 20°)

$\Rightarrow$ cosec (90° – 4A) = cosec (A – 20°)

$\Rightarrow$ 90° – 4A = A – 20°

$\Rightarrow$ 5A = 110° $\Rightarrow$ A = 22°.

6. *If A, B and C are interior angles of a triangle ABC, then show that*

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}.$$

Sol. In ΔABC,

A + B + C = 180° $\Rightarrow$ B + C = 180° – A

$$\Rightarrow \frac{B+C}{2} = 90° - \frac{A}{2}$$

$$\Rightarrow \sin\left(\frac{B+C}{2}\right) = \sin\left(90° - \frac{A}{2}\right) = \cos\frac{A}{2}.$$

7. *Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.*

Sol. Consider sin 67° + cos 75°

= sin (90° – 23°) + cos (90° – 15°) = cos 23° + sin 15°.

Exerise 8.4

1. *Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.*

Sol. (*i*) $\sin A = \dfrac{1}{\text{cosec } A} = \dfrac{1}{\sqrt{1+\cot^2 A}}$

(*ii*) $\sec A = \sqrt{1+\tan^2 A} = \sqrt{1+\dfrac{1}{\cot^2 A}} = \dfrac{\sqrt{\cot^2 A+1}}{\cot A}$

(*iii*) $\tan A = \dfrac{1}{\cot A}$.

2. *Write all the other trigonometric ratios of $\angle A$ in terms of sec A.*

Sol. $\sin A = \tan A \cdot \cos A = \dfrac{\sqrt{\sec^2 A - 1}}{\sec A}$; $\left[\cos A = \dfrac{1}{\sec A}\right]$

$\tan A = \sqrt{\sec^2 A - 1}$; $\cot A = \dfrac{1}{\tan A} = \dfrac{1}{\sqrt{\sec^2 A - 1}}$

$\text{cosec } A = \dfrac{1}{\sin A} = \dfrac{\sec A}{\sqrt{\sec^2 A - 1}}$.

3. *Evaluate*:

(*i*) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(*ii*) *sin 25° cos 65° + cos 25° sin 65°.*

Sol. (*i*) Consider $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$$= \frac{\sin^2(90° - 27°) + \sin^2 27°}{\cos^2 17° + \cos^2(90° - 17°)} = \frac{\cos^2 27° + \sin^2 27°}{\cos^2 17° + \sin^2 17°}$$

$= \dfrac{1}{1} = 1.$ $[\because \sin^2 \theta + \cos^2 \theta = 1]$

(*ii*) Consider sin 25° cos 65° + cos 25° sin 65°

= sin 25° cos (90° – 25°) + cos 25° sin (90° – 25°)

= sin 25° sin 25° + cos 25° cos 25°

$= \sin^2 25° + \cos^2 25° = 1.$ $[\because \sin^2 \theta + \cos^2 \theta = 1]$

4. *Choose the correct option. Justify your choice.*

(*i*) $9 \sec^2 A - 9 \tan^2 A =$

(*A*) *1* (*B*) *9* (*C*) *8* (*D*) *0*

(*ii*) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \text{cosec } \theta) =$

(*A*) *0* (*B*) *1* (*C*) *2* (*D*) *– 1*

(*iii*) *(sec A + tan A)(1 – sin A) =*

(*A*) *sec A* (*B*) *sin A* (*C*) *cosec A* (*D*) *cos A*

(*iv*) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A} =$

(*A*) $\sec^2 A$ (*B*) *– 1* (*C*) $\cot^2 A$ (*D*) $\tan^2 A$.

Sol. (*i*) (B). Consider $9 \sec^2 A - 9 \tan^2 A$

$$= 9 (\sec^2 A - \tan^2 A)$$

$$= 9 \times 1 = 9.$$

(*ii*) (C). Consider $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \text{cosec}\,\theta)$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$$

$$= \frac{(\cos\theta + \sin\theta)^2 - 1}{\cos\theta \sin\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta + 2\sin\theta . \cos\theta - 1}{\cos\theta . \sin\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\cos\theta\sin\theta} = 2.$$

(*iii*) (D). Consider $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{(1 + \sin A)}{\cos A} . (1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A.$$

(*iv*) (D). Consider $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\text{cosec}^2 A}$

$$= \frac{1}{\cos^2 A} . \sin^2 A$$

$$= \tan^2 A.$$

5. *Prove the following identities, where the angles involved are acute angles for which the expressions are defined.*

(*i*) $(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

(*ii*) $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$

(iii) $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\operatorname{cosec}\theta$

[**Hint:** *Write the expression in terms of sin θ and cos θ*]

(iv) $\dfrac{1+\sec A}{\sec A}=\dfrac{\sin^2 A}{1-\cos A}$

[**Hint:** *Simplify LHS and RHS separately*]

(v) $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$ $=\operatorname{cosec} A+\cot A$, *using the identity* $\operatorname{cosec}^2 A=1+\cot^2 A$.

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2$
$=7+\tan^2 A+\cot^2 A$

(ix) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

[**Hint:** *Simplify LHS and RHS separately*]

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A.$

Sol. (i) LHS $=(\operatorname{cosec}\theta-\cot\theta)^2$

$$=\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)^2=\frac{(1-\cos\theta)^2}{\sin^2\theta}=\frac{(1-\cos\theta)^2}{1-\cos^2\theta}$$

$$=\frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}=\frac{1-\cos\theta}{1+\cos\theta}=\text{RHS}.$$

(ii) LHS $=\dfrac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$

$$=\frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$$

$$= \frac{1 + 1 + 2\sin A}{(1 + \sin A)\cos A} = \frac{2(1 + \sin A)}{(1 + \sin A)\cos A}$$

$$= \frac{2}{\cos A} = 2 \sec A = \text{RHS}.$$

(*iii*) LHS $= \dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta}$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1 - \frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{(\sin\theta - \cos\theta)\cos\theta} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sec\theta \,.\, \cos\theta\,(\sin\theta - \cos\theta)}$$

$$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta\,(\sin\theta - \cos\theta)}$$

$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta \,.\, \cos\theta}$$

$$= \text{cosec}\,\theta \,.\, \sec\theta + 1 = \text{RHS}.$$

(*iv*) RHS $= \dfrac{\sin^2 A}{1 - \cos A} = \dfrac{1 - \cos^2 A}{1 - \cos A}$

$$= \frac{(1 + \cos A)(1 - \cos A)}{(1 - \cos A)}$$

$$= 1 + \cos A = 1 + \frac{1}{\sec A} = \frac{\sec A + 1}{\sec A} = \text{LHS}.$$

(*v*) LHS $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing each term of numerator and denominator by sin A, we have

$$= \frac{\frac{\cos A}{\sin A} - 1 + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + 1 - \frac{1}{\sin A}} = \frac{\cot A + \text{cosec } A - 1}{\cot A - \text{cosec } A + 1}$$

$$= \frac{(\cot A + \text{cosec } A) - (\text{cosec}^2 A - \cot^2 A)}{\cot A - \text{cosec } A + 1}$$

$$= \frac{(\cot A + \text{cosec } A)\{1 - (\text{cosec } A - \cot A)\}}{\cot A - \text{cosec } A + 1}$$

$$= \frac{(\cot A + \text{cosec } A)(1 - \text{cosec } A + \cot A)}{\cot A - \text{cosec } A + 1}$$

$= \cot A + \text{cosec } A =$ RHS.

(*vi*) LHS $= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A}$$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A =$ RHS.

(*vii*) LHS $= \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1 - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$

$$= \frac{\sin\theta(\sin^2\theta + \cos^2\theta - 2\sin^2\theta)}{\cos\theta(2\cos^2\theta - \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(\cos^2\theta - \sin^2\theta)}$$

$= \frac{\sin\theta}{\cos\theta} = \tan\theta =$ RHS.

(*viii*) LHS $= (\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \text{cosec}^2 A + 2 \sin A \text{ cosec } A + \cos^2 A$
$+ \sec^2 A + 2 \cos A \sec A$

$= 1 + 1 + \cot^2 A + 2 + 1 + \tan^2 A + 2$

[As $\sin\theta \cdot \text{cosec }\theta = 1$ and $\cos\theta \cdot \sec\theta = 1$]

$= 7 + \cot^2 A + \tan^2 A =$ RHS.

(*ix*) LHS $= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \frac{1-\sin^2 A}{\sin A} \times \frac{1-\cos^2 A}{\cos A}$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A \cos A$

$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

On dividing numerator and denominator by sin A cos A, we have

$= \frac{1}{\tan A + \cot A} = \text{RHS}.$

(*x*) LHS $= \frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{1}{\cos^2 A} \cdot \sin^2 A$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A = \text{RHS}.$

Now consider, LHS $= \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left\{\frac{\tan A\,(1-\tan A)}{\tan A - 1}\right\}^2 = \{-\tan A\}^2$

$= \tan^2 A = \text{RHS}.$